

Real Numbers

What You'll Learn

- **Lesson 2-1** Classify and graph rational numbers.
- **Lessons 2-2 through 2-4** Add, subtract, multiply, and divide rational numbers.
- **Lesson 2-5** Display and interpret statistical data on line graphs and stem-and-leaf plots.
- **Lesson 2-6** Determine simple probability and odds.
- **Lesson 2-7** Find square roots and compare real numbers.

Key Vocabulary

- rational number (p. 68)
- absolute value (p. 69)
- probability (p. 96)
- square root (p. 103)
- real number (p. 104)

Why It's Important

The ability to work with real numbers lays the foundation for further study in mathematics and allows you to solve a variety of real-world problems. For example, temperatures in the United States vary greatly from cold arctic regions to warm tropical regions. You can use real numbers and absolute value to compare these temperature extremes. *You will use absolute value and real numbers to compare temperatures in Lessons 2-1 and 2-2.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

For Lessons 2-1 through 2-5

Operations with Decimals and Fractions

Perform the indicated operation. (For review, see pages 798 and 799.)

- | | | | |
|--------------------------------|--------------------------------|-------------------------------------|-----------------------------------|
| 1. $2.2 + 0.16$ | 2. $13.4 - 4.5$ | 3. $6.4 \cdot 8.8$ | 4. $76.5 \div 4.25$ |
| 5. $\frac{1}{4} + \frac{2}{3}$ | 6. $\frac{1}{2} - \frac{1}{3}$ | 7. $\frac{5}{4} \cdot \frac{3}{10}$ | 8. $\frac{4}{9} \div \frac{1}{3}$ |

For Lessons 2-1 through 2-5

Evaluate Expressions

Evaluate each expression if $a = 2$, $b = \frac{1}{4}$, $x = 7$, and $y = 0.3$. (For review, see Lesson 1-2.)

- | | | | |
|-----------------------|--------------|------------------|-------------------|
| 9. $3a - 2$ | 10. $2x + 5$ | 11. $8(y + 2.4)$ | 12. $4(b + 2)$ |
| 13. $a - \frac{1}{2}$ | 14. $b + 3$ | 15. xy | 16. $y(a \div b)$ |

For Lesson 2-5

Find Mean, Median, and Mode

Find the mean, median, and mode for each set of data. (For review, see pages 818 and 819.)

- | | | |
|------------------------|----------------------------|-----------------------|
| 17. 2, 4, 7, 9, 12, 15 | 18. 23, 23, 23, 12, 12, 14 | 19. 7, 19, 2, 7, 4, 9 |
|------------------------|----------------------------|-----------------------|

For Lesson 2-7

Square Numbers

Simplify. (For review, see Lesson 1-1.)

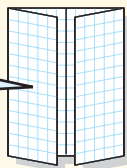
- | | | | |
|------------|-------------|----------------------------------|----------------------------------|
| 20. 11^2 | 21. 0.9^2 | 22. $\left(\frac{2}{3}\right)^2$ | 23. $\left(\frac{4}{5}\right)^2$ |
|------------|-------------|----------------------------------|----------------------------------|

FOLDABLES™ Study Organizer

Make this Foldable to collect examples and notes about operations with real numbers. Begin with a sheet of grid paper.

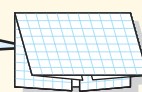
Step 1 Fold

Fold the short sides to meet in the middle.



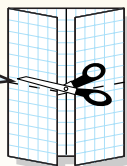
Step 2 Fold Again

Fold the top to the bottom.



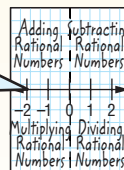
Step 3 Cut

Open. Cut along second fold to make four tabs.



Step 4 Label

Add a number line and label the tabs as shown.



Reading and Writing As you read and study the chapter, use the number line to help you solve problems. Write examples and notes under each tab.

Rational Numbers on the Number Line

What You'll Learn

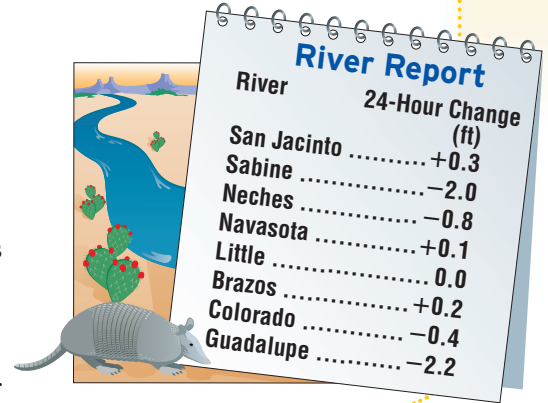
- Graph rational numbers on a number line.
- Find absolute values of rational numbers.

Vocabulary

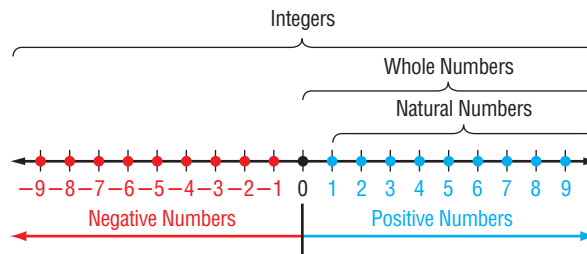
- natural number
- whole number
- integers
- positive number
- negative number
- rational number
- infinity
- graph
- coordinate
- absolute value

How can you use a number line to show data?

A river's level rises and falls depending on rainfall and other conditions. The table shows the percent of change in river depths for various rivers in Texas over a 24-hour period. You can use a number line to graph these values and compare the changes in each river.



GRAPH RATIONAL NUMBERS A number line can be used to show the sets of **natural numbers**, **whole numbers**, and **integers**. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.



Study Tip

Number Line

Although only a portion of the number line is shown, the arrowheads indicate that the line and the set continue to **infinity**, which means that they never end.

Another set of numbers you can display on a number line is the set of rational numbers. A **rational number** is any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Some examples of rational numbers are shown below.

$$\frac{1}{2} \quad \frac{-2}{3} \quad \frac{17}{5} \quad \frac{15}{-3} \quad \frac{-14}{-11} \quad \frac{3}{1}$$

A rational number can also be expressed as a decimal that terminates, or as a decimal that repeats indefinitely.

$$0.5 \quad -0.\bar{3} \quad 3.4 \quad 2.6767\dots \quad -5 \quad 1.\bar{27} \quad -1.23568994141\dots$$

Concept Summary		Rational Numbers
Natural Numbers	{1, 2, 3, ...}	
Whole Numbers	{0, 1, 2, 3, ...}	
Integers	{..., -2, -1, 0, 1, 2, ...}	
Rational Numbers	numbers expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$	

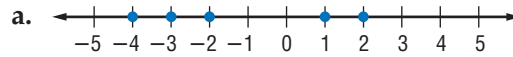
Later in this chapter, you will be introduced to numbers that are not rational.



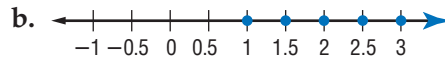
To **graph** a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the **coordinate** of that point.

Example 1 Identify Coordinates on a Number Line

Name the coordinates of the points graphed on each number line.



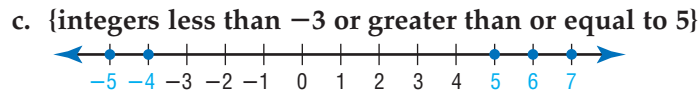
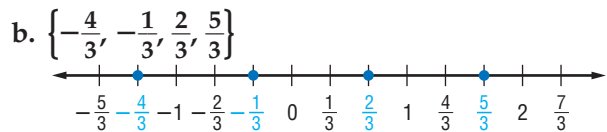
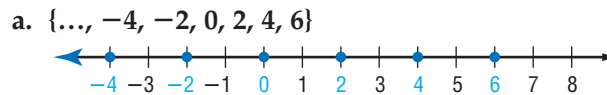
The dots indicate each point on the graph.
The coordinates are $\{-4, -3, -2, 1, 2\}$.



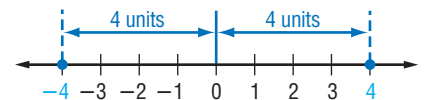
The bold arrow on the right means that the graph continues indefinitely in that direction. The coordinates are $\{1, 1.5, 2, 2.5, 3, \dots\}$.

Example 2 Graph Numbers on a Number Line

Graph each set of numbers.



ABSOLUTE VALUE On a number line, 4 is four units from zero in the positive direction, and -4 is four units from zero in the negative direction. This number line illustrates the meaning of **absolute value**.



Key Concept

Absolute Value

- **Words** The absolute value of any number n is its distance from zero on a number line and is written as $|n|$.
- **Examples** $|-4| = 4$ $|4| = 4$

Since distance cannot be less than zero, absolute values are always greater than or equal to zero.

Study Tip

Reading Math
 $|-7| = 7$ is read
the absolute value of
negative 7 equals 7.

Example 3 Absolute Value of Rational Numbers

Find each absolute value.

- a. $|-7|$
 -7 is seven units from zero in the negative direction.
 $|-7| = 7$



b. $\left| \frac{7}{9} \right|$

$\frac{7}{9}$ is seven-ninths unit from zero in the positive direction.

$$\left| \frac{7}{9} \right| = \frac{7}{9}$$

You can also evaluate expressions involving absolute value. The absolute value bars serve as grouping symbols.

Example 4 Expressions with Absolute Value

Evaluate $15 - |x + 4|$ if $x = 8$.

$$\begin{aligned} 15 - |x + 4| &= 15 - |8 + 4| && \text{Replace } x \text{ with } 8. \\ &= 15 - |12| && 8 + 4 = 12 \\ &= 15 - 12 && |12| = 12 \\ &= 3 && \text{Simplify.} \end{aligned}$$

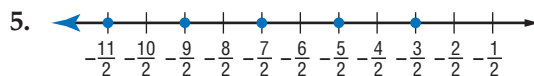
Check for Understanding

Concept Check

1. Tell whether the statement is *sometimes*, *always*, or *never* true.
An integer is a rational number.
2. Explain the meaning of absolute value.
3. **OPEN ENDED** Give an example where absolute values are used in a real-life situation.

Guided Practice

Name the coordinates of the points graphed on each number line.



Graph each set of numbers.

6. $\{-4, -2, 1, 5, 7\}$
7. $\{-2.8, -1.5, 0.2, 3.4\}$
8. $\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{5}{3}\}$
9. {integers less than or equal to -4 }

Find each absolute value.

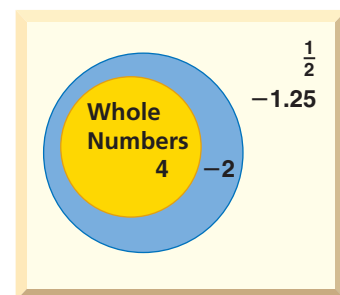
10. $|-2|$
11. $|18|$
12. $|2.5|$
13. $|\frac{-5}{6}|$

Evaluate each expression if $x = 18$, $y = 4$, and $z = -0.76$.

14. $57 - |x + 34|$
15. $19 + |21 - y|$
16. $|z| - 0.26$

Application

17. **NUMBER THEORY** Copy the Venn diagram at the right. Label the remaining sets of numbers. Then place the numbers $-3, -13, 0, 53, \frac{2}{3}, -\frac{1}{5}, 0.33, 40, 2.98, -49.98$, and $-\frac{5}{2}$ in the most specific categories.



Practice and Apply

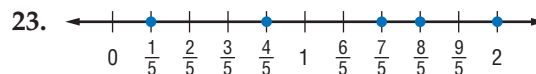
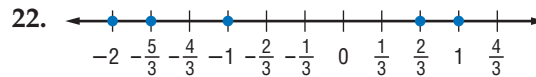
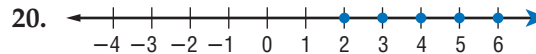
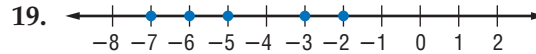
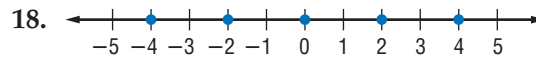
Homework Help

For Exercises	See Examples
18–23	1
24–33	2
34–41	3
42–44, 58, 59	2, 3
45–56	4

Extra Practice

See page 823.

Name the coordinates of the points graphed on each number line.



Graph each set of numbers.

- | | |
|---|--|
| 24. $\{-4, -2, -1, 1, 3\}$ | 25. $\{0, 2, 5, 6, 9\}$ |
| 26. $\{-5, -4, -3, -2, \dots\}$ | 27. $\{\dots, -2, 0, 2, 4, 6\}$ |
| 28. $\{-8.4, -7.2, -6.0, -4.8\}$ | 29. $\{-2.4, -1.6, -0.8, 3.2, \dots\}$ |
| 30. $\{\dots, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \dots\}$ | 31. $\{-3\frac{2}{5}, -2\frac{1}{5}, -1\frac{4}{5}, -\frac{4}{5}, 1\}$ |
| 32. $\{\text{integers less than } -7 \text{ or greater than } -1\}$ | |
| 33. $\{\text{integers greater than } -5 \text{ and less than } 9\}$ | |

Find each absolute value.

- | | | | |
|-------------|--------------|------------------------|-----------------------|
| 34. $ -38 $ | 35. $ 10 $ | 36. $ 97 $ | 37. $ -61 $ |
| 38. $ 3.9 $ | 39. $ -6.8 $ | 40. $ \frac{-23}{56} $ | 41. $ \frac{35}{80} $ |

POPULATION For Exercises 42–44, refer to the table below.

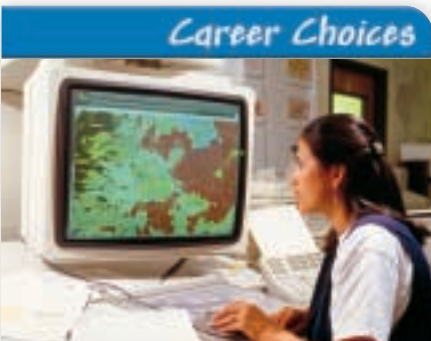
Population of Various Counties, 1990–1999			
County	Percent Change	County	Percent Change
Kings, NY	-1.4	Wayne, MI	-0.2
Los Angeles, CA	5.3	Philadelphia, PA	-10.6
Cuyahoga, OH	-2.9	Suffolk, NY	4.7
Santa Clara, CA	10.0	Alameda, CA	8.5
Cook, IL	1.7	New York, NY	4.3

Source: *The World Almanac*

42. Use a number line to order the percent of change from least to greatest.
 43. Which population had the greatest percent increase or decrease? Explain.
 44. Which population had the least percent increase or decrease? Explain.

Evaluate each expression if $a = 6$, $b = \frac{2}{3}$, $c = \frac{5}{4}$, $x = 12$, $y = 3.2$, and $z = -5$.

- | | | |
|--|-----------------------------|---|
| 45. $48 + x - 5 $ | 46. $25 + 17 + x $ | 47. $ 17 - a + 23$ |
| 48. $ 43 - 4a + 51$ | 49. $ z + 13 - 4$ | 50. $28 - 13 + z $ |
| 51. $6.5 - 8.4 - y $ | 52. $7.4 + y - 2.6 $ | 53. $\frac{1}{6} + \left b - \frac{7}{12}\right $ |
| 54. $\left(b + \frac{1}{2}\right) - \left -\frac{5}{6}\right $ | 55. $ c - 1 + \frac{2}{5}$ | 56. $ -c + \left(2 + \frac{1}{2}\right)$ |
57. **CRITICAL THINKING** Find all values for x if $|x| = -|x|$.



Career Choices

Demographer

A demographer analyses the size, nature, and movement of human populations. Many demographers specialize in one area such as health, housing, education, agriculture, or economics.

Online Research

For information about a career as a demographer, visit www.algebra1.com/careers

More About . . .



Weather

The lowest temperature ever recorded in the world was -129°F at the Soviet Antarctica station of Vostok.

Source: *The World Almanac*

• **WEATHER** For Exercises 58 and 59, use the table at the right and the information at the left.

58. Draw a number line and graph the set of numbers that represents the low temperatures for these cities.
59. Write the absolute value of the low temperature for each city.

City	Low Temperature ($^{\circ}\text{F}$)
Bismarck, ND	-11
Caribou, ME	-5
Chicago, IL	-4
Fairbanks, AK	-9
International Falls, MN	-13
Kansas City, MO	7
Sacramento, CA	34
Shreveport, LA	33

Source: *The World Almanac*



60. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use a number line to show data?

Include the following in your answer:

- an explanation of how to choose the range for a number line, and
- an explanation of how to tell which river had the greatest increase or decrease.

61. Which number is a natural number?
- (A) -2.5 (B) $5 - |5|$ (C) $-|3 + 5|$ (D) $|-8| - 2$
62. Which sentence is *not* true?
- (A) All natural numbers are whole numbers.
 (B) Natural numbers are positive numbers.
 (C) Every whole number is a natural number.
 (D) Zero is neither positive nor negative.

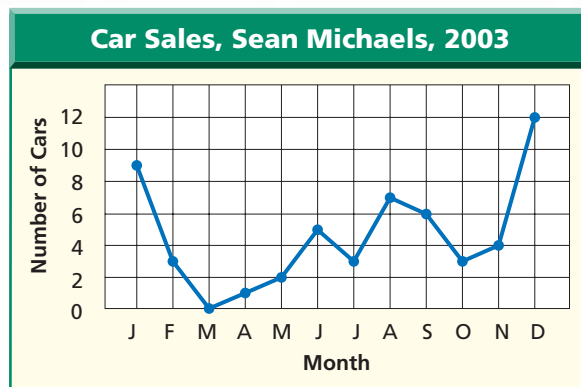
Standardized Test Practice

(A) (B) (C) (D)

Maintain Your Skills

Mixed Review SALES For Exercises 63–65, refer to the graph. (*Lesson 1-9*)

63. In which month did Mr. Michaels have the greatest sales?
64. Between which two consecutive months did the greatest change in sales occur?
65. In which months were sales equal?



66. **ENTERTAINMENT** Juanita has the volume on her stereo turned up. When her telephone rings, she turns the volume down. After she gets off the phone, she returns the volume to its previous level. Sketch a reasonable graph to show the volume of Juanita's stereo during this time. (*Lesson 1-8*)

Simplify each expression. (*Lesson 1-6*)

67. $8x + 2y + x$ 68. $7(5a + 3b) - 4a$ 69. $4[1 + 4(5x + 2y)]$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each sum or difference. (*To review addition and subtraction of fractions, see pages 798 and 799.*)

70. $\frac{3}{8} + \frac{1}{8}$ 71. $\frac{7}{12} - \frac{3}{12}$ 72. $\frac{7}{10} + \frac{1}{5}$ 73. $\frac{3}{8} + \frac{2}{3}$
74. $\frac{5}{6} + \frac{1}{2}$ 75. $\frac{3}{4} - \frac{1}{3}$ 76. $\frac{9}{15} - \frac{1}{2}$ 77. $\frac{7}{9} - \frac{7}{18}$

Adding and Subtracting Rational Numbers

What You'll Learn

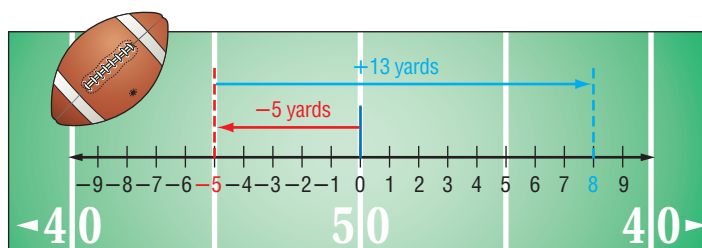
- Add integers and rational numbers.
- Subtract integers and rational numbers.

Vocabulary

- opposites
- additive inverses

How can a number line be used to show a football team's progress?

In one series of plays during Super Bowl XXXV, the New York Giants received a five-yard penalty before completing a 13-yard pass.



The number line shows the yards gained during this series of plays. The total yards gained was 8 yards.

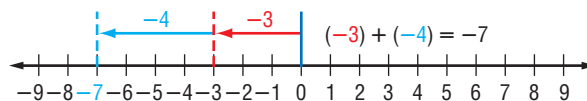
ADD RATIONAL NUMBERS The number line above illustrates how to add integers on a number line. You can use a number line to add any rational numbers.

Example 1 Use a Number Line to Add Rational Numbers

Use a number line to find each sum.

a. $-3 + (-4)$

Step 1 Draw an arrow from 0 to -3 .

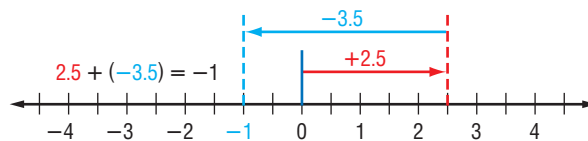


Step 2 Then draw a second arrow 4 units to the left to represent adding -4 .

Step 3 The second arrow ends at the sum -7 . So, $-3 + (-4) = -7$.

b. $2.5 + (-3.5)$

Step 1 Draw an arrow from 0 to 2.5.



Step 2 Then draw a second arrow 3.5 units to the left.

Step 3 The second arrow ends at the sum -1 . So, $2.5 + (-3.5) = -1$.

You can use absolute value to add rational numbers.

Same Signs		Different Signs	
$++$ $3 + 5 = 8$	$--$ $-3 + (-5) = -8$	$+ -$ $3 + (-5) = -2$	$- +$ $-3 + 5 = 2$
<p>3 and 5 are positive, so the sum is positive.</p>	<p>-3 and -5 are negative, so the sum is negative.</p>	<p>Since -5 has the greater absolute value, the sum is negative.</p>	<p>Since 5 has the greater absolute value, the sum is positive.</p>

The examples above suggest the following rules for adding rational numbers.

Key Concept

Addition of Rational Numbers

- To add rational numbers with the *same sign*, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different signs*, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.

Example 2 Add Rational Numbers

Find each sum.

a. $-11 + (-7)$

$$\begin{aligned} -11 + (-7) &= -(|-11| + |-7|) \\ &= -(11 + 7) \\ &= -18 \end{aligned}$$

Both numbers are negative, so the sum is negative.

b. $\frac{7}{16} + \left(-\frac{3}{8}\right)$

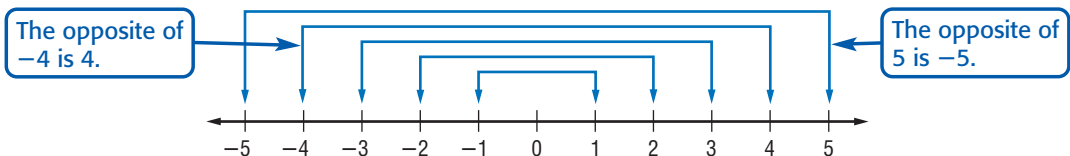
$$\begin{aligned} \frac{7}{16} + \left(-\frac{3}{8}\right) &= \frac{7}{16} + \left(-\frac{6}{16}\right) \\ &= +\left(\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right|\right) \\ &= +\left(\frac{7}{16} - \frac{6}{16}\right) \\ &= \frac{1}{16} \end{aligned}$$

The LCD is 16. Replace $-\frac{3}{8}$ with $-\frac{6}{16}$.

Subtract the absolute values.

Since the number with the greater absolute value is $\frac{7}{16}$, the sum is positive.

SUBTRACT RATIONAL NUMBERS Every positive rational number can be paired with a negative rational number. These pairs are called **opposites**.



Study Tip

Additive Inverse

Since $0 + 0 = 0$, zero is its own additive inverse.

A number and its opposite are **additive inverses** of each other. When you add two opposites, the sum is always 0.

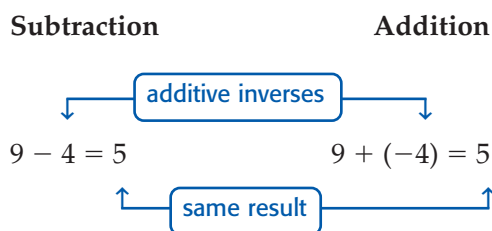


Key Concept

Additive Inverse Property

- **Words** The sum of a number and its additive inverse is 0.
- **Symbols** For every number a , $a + (-a) = 0$.
- **Examples** $2 + (-2) = 0$ $-4.25 + 4.25 = 0$ $\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Additive inverses can be used when you subtract rational numbers.



This example suggests that subtracting a number is equivalent to adding its inverse.

Key Concept

Subtraction of Rational Numbers

- **Words** To subtract a rational number, add its additive inverse.
- **Symbols** For any numbers a and b , $a - b = a + (-b)$.
- **Examples** $8 - 15 = 8 + (-15)$ or -7
 $-7.6 - 12.3 = -7.6 + (-12.3)$ or -19.9

Career Choices



Stockbroker

Stockbrokers perform various duties, including buying or selling stocks, bonds, mutual funds, or other financial products for an investor.

Online Research

For information about a career as a stockbroker, visit: www.algebra1.com/careers

Example 3 Subtract Rational Numbers to Solve a Problem

- **STOCKS** During a five-day period, a telecommunications company's stock price went from \$17.82 to \$15.36 per share. Find the change in the price of the stock.

Explore The stock price began at \$17.82 and ended at \$15.36. You need to determine the change in price for the week.

Plan Subtract to find the change in price.

$$\begin{array}{r} \text{ending price} \\ 15.36 \\ \text{minus} \\ - \\ \text{beginning price} \\ 17.82 \end{array}$$

Solve $15.36 - 17.82 = 15.36 + (-17.82)$ To subtract 17.82, add its inverse.
 $= -(|-17.82| - |15.36|)$ Subtract the absolute values.
 $= -(17.82 - 15.36)$ The absolute value of -17.82 is greater, so the result is negative.
 $= -2.46$

The price of the stock changed by $-\$2.46$.

Examine The problem asks for the change in a stock's price from the beginning of a week to the end. Since the change was negative, the price dropped. This makes sense since the ending price is less than the beginning price.

Check for Understanding

- Concept Check**
- OPEN ENDED** Write a subtraction expression using rational numbers that has a difference of $-\frac{2}{5}$.
 - Describe** how to subtract real numbers.
 - FIND THE ERROR** Gabriella and Nick are subtracting fractions.

Gabriella

$$\begin{aligned} \left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) &= \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right) \\ &= \left(-\frac{4}{9}\right) + \left(\frac{6}{9}\right) \\ &= \left(\frac{6}{9} - \frac{4}{9}\right) \\ &= \frac{2}{9} \end{aligned}$$

Nick

$$\begin{aligned} \left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) &= \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right) \\ &= \left(-\frac{4}{9}\right) + \left(-\frac{6}{9}\right) \\ &= -\left(\frac{6}{9} + \frac{4}{9}\right) \\ &= -\frac{10}{9} \end{aligned}$$

Who is correct? Explain your reasoning.

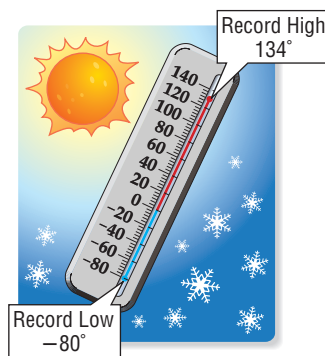
Guided Practice Find each sum.

- $-15 + (-12)$
- $-4.62 + (-12.81)$
- $-24 + (-45)$
- $\frac{4}{7} + \left(-\frac{1}{2}\right)$
- $38.7 + (-52.6)$
- $-\frac{5}{12} + \frac{8}{15}$

Find each difference.

- $18 - 23$
- $-32.25 - (-42.5)$
- $12.7 - (-18.4)$
- $-\frac{2}{9} - \frac{3}{10}$
- $(-3.86) - 1.75$
- $\left(-\frac{7}{10}\right) - \left(-\frac{11}{12}\right)$

- Application**
- WEATHER** The highest recorded temperature in the United States was in Death Valley, California, while the lowest temperature was recorded at Prospect Creek, Alaska. What is the difference between these two temperatures?



Practice and Apply

Homework Help

For Exercises	See Examples
17–38	1, 2
39–62	3

Extra Practice

See page 823.

Find each sum.

- $-8 + 13$
- $80 + (-102)$
- $-1.6 + (-3.8)$
- $-7.007 + 4.8$
- $\frac{6}{7} + \frac{2}{3}$
- $-\frac{2}{5} + \frac{17}{20}$
- $-8 + 13$
- $-11 + 19$
- $-77 + (-46)$
- $-32.4 + (-4.5)$
- $43.2 + (-57.9)$
- $\frac{3}{18} + \frac{6}{17}$
- $-\frac{4}{15} + \left(-\frac{9}{16}\right)$
- $41 + (-63)$
- $-92 + (-64)$
- $-38.9 + 24.2$
- $38.7 + (-61.1)$
- $-\frac{4}{11} + \frac{3}{5}$
- $-\frac{16}{40} + \left(-\frac{13}{20}\right)$
- Find the sum of $4\frac{1}{8}$ and $-1\frac{1}{2}$.
- Find the sum of $1\frac{17}{50}$ and $-3\frac{17}{25}$.

37. **GAMES** Sarah was playing a computer trivia game. Her scores for round one were +100, +200, +500, -300, +400, and -500. What was her total score at the end of round one?
38. **FOOTBALL** The Northland Vikings' offense began a drive from their 20-yard line. They gained 6 yards on the first down, lost 8 yards on the second down, then gained 3 yards on third down. What yard line were they on at fourth down?

Find each difference.

39. $-19 - 8$ 40. $16 - (-23)$ 41. $9 - (-24)$
 42. $12 - 34$ 43. $22 - 41$ 44. $-9 - (-33)$
 45. $-58 - (-42)$ 46. $79.3 - (-14.1)$ 47. $1.34 - (-0.458)$
 48. $-9.16 - 10.17$ 49. $67.1 - (-38.2)$ 50. $72.5 - (-81.3)$
 51. $-\frac{1}{6} - \frac{2}{3}$ 52. $\frac{1}{2} - \frac{4}{5}$ 53. $-\frac{7}{8} - \left(-\frac{3}{16}\right)$
 54. $-\frac{1}{12} - \left(-\frac{3}{4}\right)$ 55. $2\frac{1}{4} - 6\frac{1}{3}$ 56. $5\frac{3}{10} - 1\frac{31}{50}$

More About . . .



Golf

In the United States, there are more than 16,000 golf courses played by 26 million people each year.

Source: Encarta Online

GOLF For Exercises 57–59, use the following information.

In golf, scores are based on *par*. Par 72 means that a golfer should hit the ball 72 times to complete 18 holes of golf. A score of 67, or 5 under par, is written as -5. A score of 3 over par is written as +3. At the Masters Tournament (par 72) in April, 2001, Tiger Woods shot 70, 66, 68, and 68 during four rounds of golf.

57. Use integers to write his score for each round as over or under par.
 58. Add the integers to find his overall score.
 59. Was his score under or over par? Would you want to have his score? Explain.



Online Research Data Update Find the most recent winner of the Masters Tournament. What integer represents the winner's score for each round as over or under par? What integer represents the winner's overall score? Visit www.algebra1.com/data_update to learn more.

STOCKS For Exercises 60–62, refer to the table that shows the weekly closing values of the stock market for an eight-week period.

Weekly Dow Jones Industrial Average (April – May 2000)			
End of Week	Closing Value	End of Week	Closing Value
1	9791.09	5	10,951.24
2	10,126.94	6	10,821.31
3	10,579.85	7	11,301.74
4	10,810.05	8	11,257.24

Source: *The Wall Street Journal*

60. Find the change in value from week 1 to week 8.
 61. Which week had the greatest change from the previous week?
 62. Which week had the least change from the previous week?
63. **CRITICAL THINKING** Tell whether the equation $x + |x| = 0$ is *always*, *sometimes*, or *never* true. Explain.



64. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a number line be used to show a football team's progress?

Include the following in your answer:

- an explanation of how you could use a number line to determine the yards gained or lost by the Giants on their next three plays, and
- a description of how to determine the total yards gained or lost without using a number line.



65. What is the value of n in $-57 - n = -144$?
 (A) -201 (B) 201 (C) -87 (D) 87
66. Which expression is equivalent to $5 - (-8)$?
 (A) $(-5) + 8$ (B) $8 + 5$ (C) $8 - 5$ (D) $5 - 8$

Maintain Your Skills

Mixed Review Evaluate each expression if $x = 4.8$, $y = -7.4$, and $z = 10$. (Lesson 2-1)

67. $12.2 + |8 - x|$ 68. $|y| + 9.4 - 3$ 69. $24.2 - |18.3 - z|$

For Exercises 70 and 71, refer to the graph. (Lesson 1-9)

70. If you wanted to make a circle graph of the data, what additional category would you have to include so that the circle graph would not be misleading?
71. Construct a circle graph that displays the data accurately.

Find the solution sets for each inequality if the replacement sets are $A = \{2, 3, 4, 5, 6\}$, $B = \{0.3, 0.4, 0.5, 0.6, 0.7\}$, and $C = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}\}$. (Lesson 1-3)

72. $b + 1.3 \geq 1.8$
73. $3a - 5 > 7$
74. $c + \frac{1}{2} < 2\frac{1}{4}$

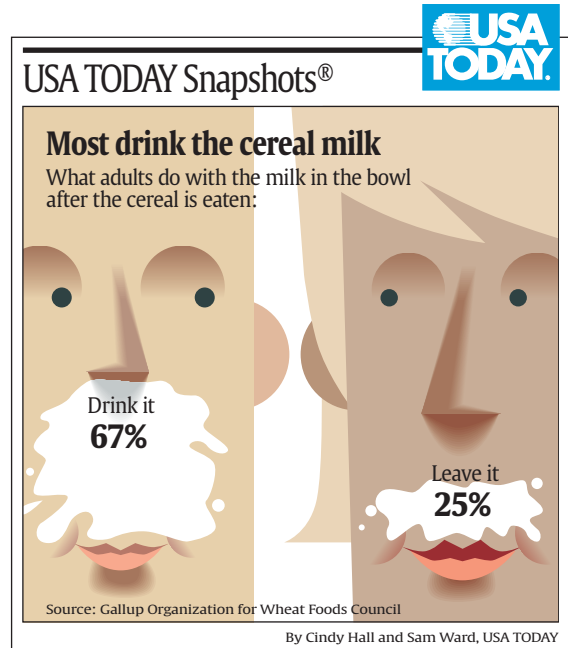
Write an algebraic expression for each verbal phrase. (Lesson 1-1)

75. eight less than the square of q 76. 37 less than 2 times a number k

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each product.
 (To review **multiplication of fractions**, see pages 800 and 801.)

77. $\frac{1}{2} \cdot \frac{2}{3}$ 78. $\frac{1}{4} \cdot \frac{2}{5}$ 79. $\frac{3}{4} \cdot \frac{5}{6}$
80. $4 \cdot \frac{3}{5}$ 81. $8 \cdot \frac{5}{8}$ 82. $\frac{7}{9} \cdot 12$



2-3

Multiplying Rational Numbers


What You'll Learn

- Multiply integers.
- Multiply rational numbers.

How do consumers use multiplication of rational numbers?

Stores often offer coupons to encourage people to shop in their stores. The receipt shows a purchase of four CDs along with four coupons for \$1.00 off each CD. How could you determine the amount saved by using the coupons?

CD SHOP	
CD.....	13.99
CD.....	12.99
CD.....	14.99
CD.....	14.99
COUPON.....	-1.00
COUPON.....	-1.00
COUPON.....	-1.00
COUPON.....	-1.00
TAX.....	0.31
TOTAL DUE.....	53.27
CASH.....	55.00
CHANGE.....	1.73



MULTIPLY INTEGERS One way to find the savings from the coupons is to use repeated addition.

$$-\$1.00 + (-\$1.00) + (-\$1.00) + (-\$1.00) = -\$4.00$$

An easier way to find the savings would be to multiply $-\$1.00$ by 4.

$$4(-\$1.00) = -\$4.00$$

Suppose the coupons were expired and had to be removed from the total. You can represent this by multiplying $-\$1.00$ by -4 .

$$(-4)(-\$1.00) = \$4.00$$

In other words, \$4.00 would be added back to the total.

These examples suggest the following rules for multiplying integers.

Study Tip

Multiplying Integers

When multiplying integers, if there are an even number of negative integers, the product is positive. If there are an odd number of negative integers, the product is negative.

Key Concept

Multiplication of Integers

- **Words** The product of two numbers having the *same sign* is positive. The product of two numbers having *different signs* is negative.
- **Examples** $(-12)(-7) = 84$ same signs \rightarrow positive product
 $15(-8) = -120$ different signs \rightarrow negative product

Example 1 Multiply Integers

Find each product.

a. $4(-5)$

$$4(-5) = -20 \quad \text{different signs} \rightarrow \text{negative product}$$

b. $(-12)(-14)$

$$(-12)(-14) = 168 \quad \text{same signs} \rightarrow \text{positive product}$$

You can simplify expressions by applying the rules of multiplication.

Example 2 Simplify Expressions

Simplify the expression $4(-3y) - 15y$.

$$\begin{aligned} 4(-3y) - 15y &= 4(-3)y - 15y && \text{Associative Property } (\times) \\ &= -12y - 15y && \text{Substitution} \\ &= (-12 - 15)y && \text{Distributive Property} \\ &= -27y && \text{Simplify.} \end{aligned}$$

MULTIPLY RATIONAL NUMBERS Multiplying rational numbers is similar to multiplying integers.

Example 3 Multiply Rational Numbers

Find $\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right)$.

$$\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right) = -\frac{9}{32} \quad \text{different signs} \rightarrow \text{negative product}$$

Example 4 Multiply Rational Numbers to Solve a Problem

BASEBALL Fenway Park, home of the Boston Red Sox, is the oldest ball park in professional baseball. It has a seating capacity of about 34,000. Determine the approximate total ticket sales for a sold-out game.

To find the approximate total ticket sales, multiply the number of tickets sold by the average price.

$$34,000 \cdot 24.05 = 817,770$$

same signs \rightarrow positive product

The total ticket sales for a sold-out game are about \$817,770.

USA TODAY

USA TODAY Snapshots®

Baseball ticket inflation

The average 1999 major league ticket was up 10% to \$14.91. The price was up 72.6% since 1991 vs. an 18.7% rise in the Consumer Price Index. Most, least expensive teams:

COSTLIEST			
	Boston (+16.6%) \$24.05	N.Y. Yankees (+13.8%) \$23.33	Texas (+20.9%) \$19.93
CHEAPEST			
	Minnesota (+2.9%) \$8.46	Montreal¹ (-6%) \$9.38	Cincinnati (+16%) \$9.71
	<small>1 - Only Montreal, Oakland and Tampa Bay cut prices for 1999</small>		

Source: Team Marketing Report

By Scott Boeck and Marcy E. Mullins, USA TODAY



Log on for:

- Updated data
 - More activities on writing equations
- www.algebra1.com/usa_today

You can evaluate expressions that contain rational numbers.

Example 5 Evaluate Expressions

Evaluate $n^2\left(-\frac{5}{8}\right)$ if $n = -\frac{2}{5}$.

$$\begin{aligned} n^2\left(-\frac{5}{8}\right) &= \left(-\frac{2}{5}\right)^2\left(-\frac{5}{8}\right) && \text{Substitution} \\ &= \left(\frac{4}{25}\right)\left(-\frac{5}{8}\right) && \left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) \text{ or } \frac{4}{25} \\ &= -\frac{20}{200} \text{ or } -\frac{1}{10} && \text{different signs} \rightarrow \text{negative product} \end{aligned}$$



In Lesson 1-4, you learned about the Multiplicative Identity Property, which states that any number multiplied by 1 is equal to the number. Another important property is the Multiplicative Property of -1 .

Key Concept

Multiplicative Property of -1

- **Words** The product of any number and -1 is its additive inverse.
- **Symbols** For any number a , $-1(a) = a(-1) = -a$.
- **Examples** $(-1)(4) = (4)(-1) = -4$ $(-1)(-2.3) = (-2.3)(-1) = 2.3$

Check for Understanding

- Concept Check**
1. **List** the conditions under which the product ab is negative. Give examples to support your answer.
 2. **OPEN ENDED** Describe a real-life situation in which you would multiply a positive rational number by a negative rational number. Write a corresponding multiplication expression.
 3. **Explain** why the product of two negative numbers is positive.

Guided Practice Find each product.

4. $(-6)(3)$
5. $(5)(-8)$
6. $(4.5)(2.3)$
7. $(-8.7)(-10.4)$
8. $\left(\frac{5}{3}\right)\left(-\frac{2}{7}\right)$
9. $\left(-\frac{4}{9}\right)\left(\frac{7}{15}\right)$

Simplify each expression.

10. $5s(-6t)$
11. $6x(-7y) + (-15xy)$

Evaluate each expression if $m = -\frac{2}{3}$, $n = \frac{1}{2}$, and $p = -3\frac{3}{4}$.

12. $6m$
13. np
14. $n^2(m + 2)$

- Application**
15. **NATURE** The average worker honeybee makes about $\frac{1}{12}$ teaspoon of honey in its lifetime. How much honey do 675 honeybees make?

Practice and Apply

Homework Help

For Exercises	See Examples
16–33	1, 3
34–39	2
40, 41	4
42–49	5
50–54	4

Extra Practice

See page 823.

Find each product.

16. $5(18)$
17. $8(22)$
18. $-12(15)$
19. $-24(8)$
20. $-47(-29)$
21. $-81(-48)$
22. $\left(\frac{4}{5}\right)\left(\frac{3}{8}\right)$
23. $\left(\frac{5}{12}\right)\left(\frac{4}{9}\right)$
24. $\left(-\frac{3}{5}\right)\left(\frac{5}{6}\right)$
25. $\left(-\frac{2}{5}\right)\left(\frac{6}{7}\right)$
26. $\left(-3\frac{1}{5}\right)\left(-7\frac{1}{2}\right)$
27. $\left(-1\frac{4}{5}\right)\left(-2\frac{1}{2}\right)$
28. $7.2(0.2)$
29. $6.5(0.13)$
30. $(-5.8)(2.3)$
31. $(-0.075)(6.4)$
32. $\frac{3}{5}(-5)(-2)$
33. $\frac{2}{11}(-11)(-4)$

Simplify each expression.

34. $6(-2x) - 14x$
35. $5(-4n) - 25n$
36. $5(2x - x)$
37. $-7(3d + d)$
38. $-2a(-3c) + (-6y)(6r)$
39. $7m(-3n) + 3s(-4t)$



57. Which expression can be simplified as $-8xy$?
 (A) $2y - 4x$ (B) $-2x(4y)$ (C) $(-4)^2xy$ (D) $-4x(-2y)$
58. Find the value of m if $m = -2ab$, $a = -4$, and $b = 6$.
 (A) 8 (B) 48 (C) 12 (D) -48

Maintain Your Skills

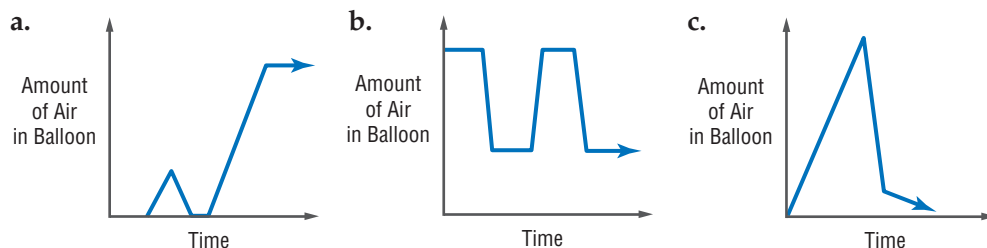
Mixed Review Find each sum or difference. (Lesson 2-2)

59. $-6.5 + (-5.6)$ 60. $\frac{4}{5} + (-\frac{3}{4})$ 61. $42 - (-14)$ 62. $-14.2 - 6.7$

Graph each set of numbers on a number line. (Lesson 2-1)

63. $\{\dots, -3, -1, 1, 3, 5\}$ 64. $\{-2.5, -1.5, 0.5, 4.5\}$ 65. $\{-1, -\frac{1}{3}, \frac{2}{3}, 2\}$

66. Identify the graph below that best represents the following situation. Brandon has a deflated balloon. He slowly fills the balloon up with air. Without tying the balloon, he lets it go. (Lesson 1-8)



Write a counterexample for each statement. (Lesson 1-7)

67. If $2x - 4 \geq 6$, then $x > 5$. 68. If $|a| > 3$, then $a > 3$.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each quotient.

(To review **division of fractions**, see pages 800 and 801.)

69. $\frac{5}{8} \div 2$ 70. $\frac{2}{3} \div 4$ 71. $5 \div \frac{3}{4}$ 72. $1 \div \frac{2}{5}$
73. $\frac{1}{2} \div \frac{3}{8}$ 74. $\frac{7}{9} \div \frac{5}{6}$ 75. $\frac{4}{5} \div \frac{6}{5}$ 76. $\frac{7}{8} \div \frac{2}{3}$

Practice Quiz 1

Lessons 2-1 through 2-3

1. Name the set of points graphed on the number line. (Lesson 2-1)



2. Evaluate $32 - |x + 8|$ if $x = 15$. (Lesson 2-1)

Find each sum or difference. (Lesson 2-2)

3. $-15 + 7$ 4. $27 - (-12)$ 5. $-6.05 + (-2.1)$ 6. $-\frac{3}{4} - (-\frac{2}{5})$

Find each product. (Lesson 2-3)

7. $-9(-12)$ 8. $(3.8)(-4.1)$

9. Simplify $(-8x)(-2y) + (-3y)(z)$. (Lesson 2-3)

10. Evaluate $mn + 5$ if $m = 2.5$ and $n = -3.2$. (Lesson 2-3)



Dividing Rational Numbers

What You'll Learn

- Divide integers.
- Divide rational numbers.

How can you use division of rational numbers to describe data?

Each year, many sea turtles are stranded on the Texas Gulf Coast. The number of sea turtles stranded from 1997 to 2000 and the changes in number from the previous years are shown in the table. The following expression can be used to find the *mean* change per year of the number of stranded turtles.

$$\text{mean} = \frac{(-127) + 54 + (-65)}{3}$$

Stranded Sea Turtles Texas Gulf Coast		
Year	Number of Turtles	Change
1997	523	—
1998	396	-127
1999	450	+54
2000	385	-65

Source: www.ridleyturtles.org

DIVIDE INTEGERS Since multiplication and division are inverse operations, the rule for finding the sign of the quotient of two numbers is similar to the rule for finding the sign of a product of two numbers.

Key Concept

Division of Integers

- **Words** The quotient of two numbers having the *same sign* is positive. The quotient of two numbers having *different signs* is negative.
- **Examples** $(-60) \div (-5) = 12$ same signs \rightarrow positive quotient
 $32 \div (-8) = -4$ different signs \rightarrow negative quotient

Example 1 Divide Integers

Find each quotient.

a. $-77 \div 11$

$$-77 \div 11 = -7 \quad \text{negative quotient}$$

b. $\frac{-51}{-3}$

$$\frac{-51}{-3} = -51 \div (-3) \quad \text{Divide.}$$

$$= 17 \quad \text{positive quotient}$$

When simplifying fractions, recall that the fraction bar is a grouping symbol.

Example 2 Simplify Before Dividing

Simplify $\frac{-3(-12 + 8)}{7 + (-5)}$.

$$\frac{-3(-12 + 8)}{7 + (-5)} = \frac{-3(-4)}{7 + (-5)} \quad \text{Simplify the numerator first.}$$

$$= \frac{12}{7 + (-5)} \quad \text{Multiply.}$$

$$= \frac{12}{2} \text{ or } 6 \quad \text{same signs } \rightarrow \text{ positive quotient}$$

DIVIDE RATIONAL NUMBERS The rules for dividing positive and negative integers also apply to division with rational numbers. Remember that to divide by any nonzero number, multiply by the reciprocal of that number.

Example 3 *Divide Rational Numbers*

Find each quotient.

a. $245.66 \div (-14.2)$

$245.66 \div (-14.2) = -17.3$ Use a calculator.
different signs \rightarrow negative quotient

b. $-\frac{2}{5} \div \frac{1}{4}$

$-\frac{2}{5} \div \frac{1}{4} = -\frac{2}{5} \cdot \frac{4}{1}$ Multiply by $\frac{4}{1}$, the reciprocal of $\frac{1}{4}$.
 $= -\frac{8}{5}$ or $-1\frac{3}{5}$ different signs \rightarrow negative quotient

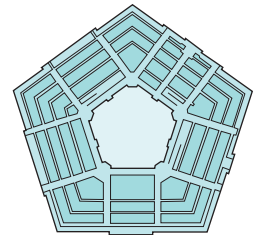
Example 4 *Divide Rational Numbers to Solve a Problem*

- ARCHITECTURE** The Pentagon in Washington, D.C., has an outside perimeter of 4608 feet. Find the length of each outside wall.

To find the length of each wall, divide the perimeter by the number of sides.

$4608 \div 5 = 921.6$ same signs \rightarrow positive quotient

The length of each outside wall is 921.6 feet.



The Pentagon

You can use the Distributive Property to simplify fractional expressions.

Example 5 *Simplify Algebraic Expressions*

Simplify $\frac{24 - 6a}{3}$.

$\frac{24 - 6a}{3} = (24 - 6a) \div 3$ The fraction bar indicates division.

$= (24 - 6a)\left(\frac{1}{3}\right)$ Multiply by $\frac{1}{3}$, the reciprocal of 3.

$= 24\left(\frac{1}{3}\right) - 6a\left(\frac{1}{3}\right)$ Distributive Property

$= 8 - 2a$ Simplify.

Example 6 *Evaluate Algebraic Expressions*

Evaluate $\frac{ab}{c^2}$ if $a = -7.8$, $b = 5.2$, and $c = -3$. Round to the nearest hundredth.

$\frac{ab}{c^2} = \frac{(-7.8)(5.2)}{(-3)^2}$ Replace a with -7.8 , b with 5.2 , and c with -3 .

$= \frac{-40.56}{9}$ Find the numerator and denominator separately.

≈ 4.51 Use a calculator. same signs \rightarrow positive quotient

More About . . .



Architecture

The Pentagon is one of the world's largest office buildings. It contains 131 stairways, 19 escalators, 13 elevators, 284 restrooms, and 691 drinking fountains.

Source: www.infoplease.com



Check for Understanding

- Concept Check**
1. Compare and contrast multiplying and dividing rational numbers.
 2. **OPEN ENDED** Find a value for x if $\frac{1}{x} > x$.
 3. Explain how to divide any rational number by another rational number.

Guided Practice Find each quotient.

4. $96 \div (-6)$
5. $-36 \div 4$
6. $-64 \div 5$
7. $64.4 \div 2.5$
8. $-\frac{2}{3} \div 12$
9. $-\frac{2}{3} \div \frac{4}{5}$

Simplify each expression.

10. $\frac{25 + 3}{-4}$
11. $\frac{-650a}{10}$
12. $\frac{6b + 18}{-2}$

Evaluate each expression if $a = 3$, $b = -4.5$, and $c = 7.5$. Round to the nearest hundredth.

13. $\frac{2ab}{-ac}$
14. $\frac{cb}{4a}$
15. $-\frac{a}{b} \div \frac{a}{c}$

- Application**
16. **ONLINE SHOPPING** During the 2000 holiday season, the sixth most visited online shopping site recorded 419,000 visitors. This is eight times as many visitors as in 1999. About how many visitors did the site have in 1999?

Practice and Apply

Homework Help

For Exercises	See Examples
17–36	1, 3
37–44	5
45, 46,	4
55–57	
47–54	6

Extra Practice

See page 824.

Find each quotient.

17. $-64 \div (-8)$
18. $-78 \div (-4)$
19. $-78 \div (-1.3)$
20. $108 \div (-0.9)$
21. $42.3 \div (-6)$
22. $68.4 \div (-12)$
23. $-23.94 \div 10.5$
24. $-60.97 \div 13.4$
25. $-32.25 \div (-2.5)$
26. $-98.44 \div (-4.6)$
27. $-\frac{1}{3} \div 4$
28. $-\frac{3}{4} \div 12$
29. $-7 \div \frac{3}{5}$
30. $-5 \div \frac{2}{7}$
31. $\frac{16}{36} \div \frac{24}{60}$
32. $-\frac{24}{56} \div \frac{31}{63}$
33. $\frac{14}{32} \div \left(-\frac{12}{25}\right)$
34. $\frac{80}{25} \div \left(-\frac{2}{3}\right)$

35. Find the quotient of -74 and $-\frac{5}{3}$.
36. Find the quotient of -156 and $-\frac{3}{8}$.

Simplify each expression.

37. $\frac{81c}{9}$
38. $\frac{105g}{5}$
39. $\frac{8r + 24}{-8}$
40. $\frac{7h + 35}{-7}$
41. $\frac{40a - 50b}{2}$
42. $\frac{42c - 18d}{3}$
43. $\frac{-8f + (-16g)}{8}$
44. $\frac{-5x + (-10y)}{5}$

45. **CRAFTS** Hannah is making pillows. The pattern states that she needs $1\frac{3}{4}$ yards of fabric for each pillow. If she has $4\frac{1}{2}$ yards of fabric, how many pillows can she make?
46. **BOWLING** Bowling centers in the United States made \$2,800,000,000 in 1990. Their receipts in 1998 were \$2,764,000,000. What was the average change in revenue for each of these 8 years? **Source:** U.S. Census Bureau



Evaluate each expression if $m = -8$, $n = 6.5$, $p = 3.2$, and $q = -5.4$.
Round to the nearest hundredth.

47. $\frac{mn}{p}$

48. $\frac{np}{m}$

49. $mq \div np$

50. $pq \div mn$

51. $\frac{n+p}{m}$

52. $\frac{m+p}{q}$

53. $\frac{m-2n}{-n+q}$

54. $\frac{p-3q}{-q-m}$

55. **BUSINESS** The president of a small business is looking at her profit/loss statement for the past year. The loss in income for the last year was \$23,985. On average, what was the loss per month last year?

• **JEWELRY** For Exercises 56 and 57, use the following information.

The gold content of jewelry is given in karats. For example, 24-karat gold is pure gold, and 18-karat gold is $\frac{18}{24}$ or 0.75 gold.

56. What fraction of 10-karat gold is pure gold? What fraction is not gold?

57. If a piece of jewelry is $\frac{2}{3}$ gold, how would you describe it using karats?

58. **CRITICAL THINKING** What is the least positive integer that is divisible by all whole numbers from 1 to 9?

59. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use division of rational numbers to describe data?

Include the following in your answer:

- an explanation of how you could use the mean of a set of data to describe changes in the data over time, and
- reasons why you think the change from year to year is not consistent.



Jewelry

The discovery of gold at Sutter's Mill early in 1848 brought more than 40,000 prospectors to California within two years.

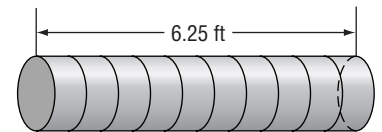
Source: www.infoplease.com

Standardized Test Practice

- (A) (B) (C) (D)

60. If the rod is cut as shown, how many inches long will each piece be?

- (A) 0.625 in. (B) 1.875 in.
(C) 5.2 in. (D) 7.5 in.



61. If $\frac{17}{3} = x$, then what is the value of $6x + 1$?

- (A) 32 (B) 33 (C) 44 (D) 35

Maintain Your Skills

Mixed Review Find each product. (Lesson 2-3)

62. $-4(11)$ 63. $-2.5(-1.2)$ 64. $\frac{1}{4}(-5)$ 65. $1.6(0.3)$

Find each difference. (Lesson 2-2)

66. $8 - (-6)$ 67. $15 - 21$ 68. $-7.5 - 4.8$ 69. $-\frac{5}{8} - (-\frac{1}{6})$

70. Name the property illustrated by $2(1.2 + 3.8) = 2 \cdot 5$.

Simplify each expression. If not possible, write *simplified*. (Lesson 1-5)

71. $8b + 12(b + 2)$ 72. $6(5a + 3b - 2b)$ 73. $3(x + 2y) - 2y$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the mean, median, and mode for each set of data. (To review *mean*, *median*, and *mode*, see pages 818 and 819.)

74. 40, 34, 40, 28, 38 75. 3, 9, 0, 2, 11, 8, 14, 3
76. 1.2, 1.7, 1.9, 1.8, 1.2, 1.0, 1.5 77. 79, 84, 81, 84, 75, 73, 80, 78



Statistics: Displaying and Analyzing Data

What You'll Learn

- Interpret and create line plots and stem-and-leaf plots.
- Analyze data using mean, median, and mode.

Vocabulary

- line plot
- frequency
- stem-and-leaf plot
- back-to-back stem-and-leaf plot
- measures of central tendency

How are line plots and averages used to make decisions?

How many people do you know with the same first name? Some names are more popular than others. The table below lists the top five most popular names for boys and girls born in each decade from 1950 to 1999.

Decade	Boys	Girls
1950-59	Michael, James, Robert, John, David	Deborah, Mary, Linda, Patricia, Susan
1960-69	Michael, John, David, James, Robert	Lisa, Deborah, Mary, Karen, Michelle
1970-79	Michael, Christopher, Jason, David, James	Jennifer, Michelle, Amy, Melissa, Kimberly
1980-89	Michael, Christopher, Matthew, Joshua, David	Jessica, Jennifer, Ashley, Sarah, Amanda
1990-99	Michael, Christopher, Matthew, Joshua, Nicholas	Ashley, Jessica, Sarah, Brittany, Emily

Source: *The World Almanac*

To help determine which names appear most frequently, these data could be displayed graphically.

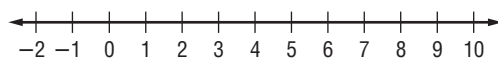
CREATE LINE PLOTS AND STEM-AND-LEAF PLOTS In some cases, data can be presented using a **line plot**. Most line plots have a number line labeled with a scale to include all the data. Then an \times is placed above a data point each time it occurs to represent the **frequency** of the data.

Example 1 Create a Line Plot

Draw a line plot for the data.

-2 4 3 2 6 10 7 4 -2 0 10 8 7 10 7 4 -1 9 -1 3

Step 1 The value of the data ranges from -2 to 10 , so construct a number line containing those points.



Step 2 Then place an \times above a number each time it occurs.



Line plots are a convenient way to organize data for comparison.

Example 2 Use a Line Plot to Solve a Problem

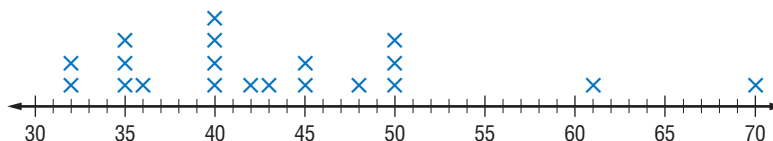
ANIMALS The speeds (mph) of 20 of the fastest land animals are listed below.

45 70 43 45 32 42 40 40 35 50
40 35 61 48 35 32 50 36 50 40

Source: *The World Almanac*

a. Make a line plot of the data.

The lowest value is 30, and the highest value is 70, so use a scale that includes those values. Place an \times above each value for each occurrence.



b. Which speed occurs most frequently?

Looking at the line plot, we can easily see that 40 miles per hour occurs most frequently.

Another way to organize and display data is by using a **stem-and-leaf plot**. In a stem-and-leaf plot, the greatest common place value is used for the *stems*. The numbers in the next greatest place value are used to form the *leaves*. In Example 2, the greatest place value is tens. Thus, 32 miles per hour would have a stem of 3 and a leaf of 2. A complete stem-and-leaf plot for the data in Example 2 is shown below.

Stem	Leaf
3	2 2 5 5 5 6
4	0 0 0 0 2 3 5 5 8
5	0 0
6	1
7	0

3 | 2 = 32
↑
key

Example 3 Create a Stem-and-Leaf Plot

Use the data below to make a stem-and-leaf plot.

108 104 86 82 80 72 70 62 64 68 84 64 98 96 98
103 87 65 83 79 97 96 112 62 80 62 83 76 66 97

The greatest common place value is tens, so the digits in the tens place are the stems.

Stem	Leaf
6	2 2 2 4 4 5 6 8
7	0 2 6 9
8	0 0 2 3 3 4 6 7
9	6 6 7 7 8 8
10	3 4 8
11	2

10 | 3 = 103

A **back-to-back stem-and-leaf plot** can be used to compare two related sets of data.



More About...

Animals

Whereas the cheetah is the fastest land animal, the fastest marine animal is the sailfish. It is capable of swimming 68 miles per hour.

Source: *The Top 10 of Everything*

Study Tip

Stem-and-Leaf Plots

A key is included on stem-and-leaf plots to indicate what the stems and leaves represent when read.

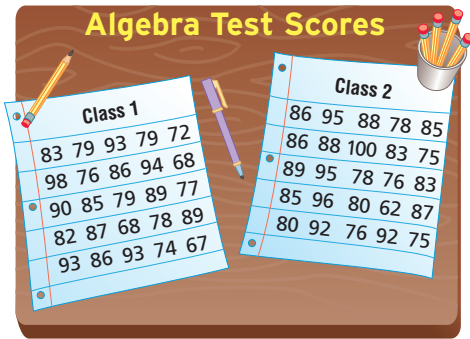


Example 4 Back-to-Back Stem-and-Leaf Plot

Mrs. Evans wants to compare recent test scores from her two algebra classes. The table shows the scores for both classes.

- a. Make a stem-and-leaf plot to compare the data.

To compare the data, we can use a back-to-back stem-and-leaf plot. Since the data represent similar measurements, the plot will share a common stem.



Class 1	Stem	Class 2
8 8 7	6	2
9 9 9 8 7 6 4 2	7	5 5 6 6 8 8
9 9 7 6 6 5 3 2	8	0 0 3 3 5 5 6 6 7 8 8 9
8 4 3 3 3 0	9	2 2 5 5 6
7 6 = 67	10	0 6 2 = 62

- b. What is the difference between the highest score in each class?

100 – 98 or 2 points

- c. Which class scored higher overall on the test?

Looking at the scores of 80 and above, we see that class 2 has a greater number of scores at or above 80 than class 1.

ANALYZE DATA When analyzing data, it is helpful to have one number that describes the set of data. Numbers known as **measures of central tendency** are often used to describe sets of data because they represent a centralized, or middle, value. Three of the most commonly used measures of central tendency are the mean, median, and mode.

When you use a measure of central tendency to describe a set of data, it is important that the measure you use best represents all of the data.

- Extremely high or low values can affect the mean, while not affecting the median or mode.
- A value with a high frequency can cause the mode to be misleading.
- Data that is clustered with a few values separate from the cluster can cause the median to be too low or too high.

Example 5 Analyze Data

Which measure of central tendency best represents the data?

Determine the mean, median, and mode.

The mean is about 0.88. *Add the data and divide by 15.*

The median is 0.82. *The middle value is 0.82.*

The mode is 0.82. *The most frequent value is 0.82.*

Stem	Leaf
7	7 8 9
8	2 2 2 2 2 3 4 4 6
9	
10	8
11	6 8 7 9 = 0.79

Either the median or the mode best represent the set of data since both measures are located in the center of the majority of the data. In this instance, the mean is too high.

Study Tip

Look Back

To review **finding mean, median, and mode**, see pages 818 and 819.

Example 6 Determine the Best Measure of Central Tendency

PRESIDENTS The numbers below show the ages of the U.S. Presidents since 1900 at the time they were inaugurated. Which measure of central tendency best represents the data?

42 51 56 55 51 54 51 60 62
43 55 56 61 52 69 64 46 54

The mean is about 54.6. Add the data and divide by 18.

The median is 54.5. The middle value is 54.5.

The mode is 51. The most frequent value is 51.

The mean or the median can be used to best represent the data. The mode for the data is too low.

Check for Understanding

Concept Check

1. Explain why it is useful to find the mean, median, and mode of a set of data.
2. Mitchell says that a line plot and a line graph are the same thing. Find a counterexample to show that he is incorrect.
3. **OPEN ENDED** Write a set of data for which the median is a better representation than the mean.

Guided Practice

4. Use the data to make a line plot.
22 19 14 15 14 21 19 16 22 19 10 15 19 14 19

For Exercises 5–7, use the list that shows the number of hours students in Mr. Ricardo’s class spent online last week.

7 4 7 11 3 1 5 10 10 0 9 4 0 14 13 4
11 3 1 12 0 9 13 14 7 6 10 5 12 0 6 5

5. Make a line plot of the data.
6. Which value occurs most frequently?
7. Does the mean, median, or mode best represent the data? Explain.
8. Use the data to make a stem-and-leaf plot.

68 66 68 88 76 71 88 93 86 64 73 80 81 72 68

For Exercises 9 and 10, use the data in the stem-and-leaf plot.

9. What is the difference between the least and greatest values?
10. Which measure of central tendency best describes the data? Explain.

Stem	Leaf
9	3 5 5
10	2 2 5 8
11	5 8 8 9 9 9
12	0 1 7 8 9 $9 3 = 9.3$

Application

BUILDINGS For Exercises 11–13, use the data below that represents the number of stories in the 25 tallest buildings in the world.

88 88 110 88 80 69 102 78 70 54 80 85
83 100 60 90 77 55 73 55 56 61 75 64 105

11. Make a stem-and-leaf plot of the data.
12. Which value occurs most frequently?
13. Does the mode best describe the set of data? Explain.



Practice and Apply

Homework Help

For Exercises	See Examples
14–18	1, 2
20–22, 28, 29, 32, 33, 35, 36	3, 4
19, 23–27, 30, 31, 34, 37	5, 6

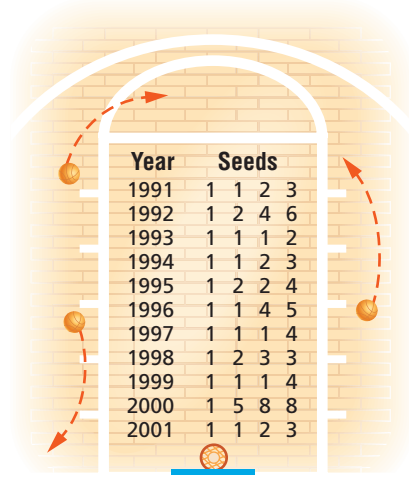
Extra Practice

See page 824.

Use each set of data to make a line plot.

14. 43 36 48 52 41 54 45 48 49 52 35 44 53 46 38 41 53
15. 1.0 –1.5 1.5 2.0 –1.5 2.1 –2.0 2.4 1.5 –1.4
2.5 1.4 –1.2 1.3 1.0 2.2 2.3 –1.2 –1.5 2.1

BASKETBALL For Exercises 16–19, use the table that shows the seeds, or rank, of the NCAA men’s basketball Final Four from 1991 to 2001.



Source: www.espn.com

16. Make a line plot of the data.
17. How many of the teams in the Final Four were *not* number 1 seeds?
18. How many teams were seeded higher than third? (*Hint*: Higher seeds have lesser numerical value.)
19. Which measure of central tendency best describes the data? Explain.

Use each set of data to make a stem-and-leaf plot.

20. 6.5 6.3 6.9 7.1 7.3 5.9 6.0 7.0 7.2 6.6 7.1 5.8
21. 31 30 28 26 22 34 26 31 47 32 18 33 26 23 18 29

WEATHER For Exercises 22–24, use the list of the highest recorded temperatures in each of the 50 states.

112	100	128	120	134	118	106	110	109	112
100	118	117	116	118	121	114	114	105	109
107	112	114	115	118	117	118	125	106	110
122	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: *The World Almanac*

22. Make a stem-and-leaf plot of the data.
23. Which temperature occurs most frequently?
24. Does the mode best represent the data? Explain.
25. **RESEARCH** Use the Internet or another source to find the total number of each CD sold over the past six months to reach number one. Which measure of central tendency best describes the average number of top selling CDs sold? Explain.

GEOLOGY For Exercises 26 and 27, refer to the stem-and-leaf plot that shows the magnitudes of earthquakes occurring in 2000 that measured at least 5.0 on the Richter scale.

26. What was the most frequent magnitude of these earthquakes?
27. Which measure of central tendency best describes this set of data? Explain.

Stem	Leaf
5	1 2 2 3 4 8 8 9 9
6	1 1 2 3 4 5 6 7 7 8
7	0 1 1 2 2 3 5 5 5 6 8 8
8	0 0 2 5 1 = 5.1

Source: National Geophysical Data Center

OLYMPICS For Exercises 28–31, use the information in the table that shows the number of medals won by the top ten countries during the 2000 Summer Olympics in Sydney, Australia.

Country	Gold	Silver	Bronze	Total
United States	40	24	33	97
Russia	32	28	28	88
China	28	16	15	59
Australia	16	25	17	58
Germany	13	17	26	56
France	13	14	11	38
Italy	14	8	13	35
Cuba	11	11	7	29
Britain	11	10	7	28
Korea	8	10	10	28

Source: www.espn.com

- Make a line plot showing the number of gold medals won by the countries.
- How many countries won fewer than 25 gold medals?
- What was the median number of gold medals won by a country?
- Is the median the best measure to describe this set of data? Explain.

CARS For Exercises 32–34, use the list of the fuel economy of various vehicles in miles per gallon.

25	28	29	30	24	28	29	31	34	30
33	47	34	43	33	36	37	29	30	30
29	26	29	22	23	19	18	20	23	21
20	20	19	16	18	21	20	19	28	20

Source: United States Environmental Protection Agency

- Make a stem-and-leaf plot of the data.
- How many of the vehicles get more than 25 miles per gallon?
- Which measure of central tendency would you use to describe the fuel economy of the vehicles? Explain your reasoning.

EDUCATION For Exercises 35–37, use the table that shows the top ten public libraries in the United States by population served.

Location	Number of Branches	Location	Number of Branches
Brooklyn, NY	59	Los Angeles County, CA	84
Broward County, FL	34	Miami, FL	30
Chicago, IL	77	New York, NY	85
Houston, TX	37	Philadelphia P	52
Los Angeles, CA	67	Queens Borough, NY	62

- Make a stem-and-leaf plot to show the number of library branches.
- Which interval has the most values?
- What is the mode of the data?
- CRITICAL THINKING** Construct a set of twelve numbers with a mean of 7, a median of 6, and a mode of 8.

More About . . .



Education

In 1848, the Boston Public Library became the first public library to allow users to borrow books and materials.

Source: The Boston Public Library



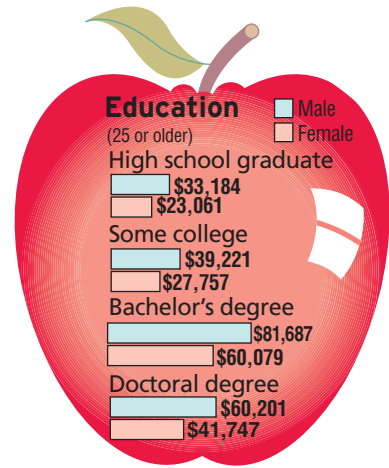
SALARIES For Exercises 39–41, refer to the bar graph that shows the median income of males and females based on education levels.

39. What are the differences between men's and women's salaries at each level of education?
40. What do these graphs say about the difference between salaries and education levels?
41. Why do you think that salaries are usually represented by the median rather than the mean?
42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are line plots and averages used to make decisions?

Include the following in your answer:

- a line plot to show how many male students in your class have the most popular names for the decade in which they were born, and
- a convincing argument that explains how you would use this information to sell personalized T-shirts.



Source: USA TODAY

Standardized Test Practice

A B C D

For Exercises 43 and 44, refer to the line plot.

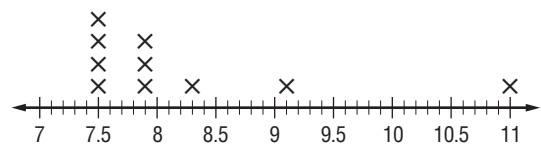
43. What is the average wingspan for these types of butterflies?

- (A) 7.6 in. (B) 7.9 in.
(C) 8.2 in. (D) 9.1 in.

44. Which sentence is *not* true?

- (A) The difference between the greatest and least wingspan is 3.5 inches.
(B) Most of the wingspans are in the 7.5 inch to 8.5 inch interval.
(C) Most of the wingspans are greater than 8 inches.
(D) The mode of the data is 7.5 inches.

Wingspan (in.) of Ten Largest Butterflies



Maintain Your Skills

Mixed Review Find each quotient. (Lesson 2-4)

45. $56 \div (-14)$ 46. $-72 \div (-12)$ 47. $-40.5 \div 3$ 48. $102 \div 6.8$

Simplify each expression. (Lesson 2-3)

49. $-2(6x) - 5x$ 50. $3x(-7y) - 4x(5y)$ 51. $5(3t - 2t) - 2(4t)$

52. Write an algebraic expression to represent the amount of money in Kara's savings account if she has d dollars and adds x dollars per week for 12 weeks. (Lesson 1-1)

Evaluate each expression if $x = 5$, $y = 16$, and $z = 9$. (Lesson 1-2)

53. $y - 3x$ 54. $xz \div 3$ 55. $2x - x + (y \div 4)$ 56. $\frac{x^2 - z}{2y}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each fraction in simplest form.

(To review *simplifying fractions*, see pages 798 and 799.)

57. $\frac{12}{18}$ 58. $\frac{54}{60}$ 59. $\frac{21}{30}$ 60. $\frac{42}{48}$
61. $\frac{32}{64}$ 62. $\frac{28}{52}$ 63. $\frac{16}{36}$ 64. $\frac{84}{90}$



Reading Mathematics

Interpreting Statistics

The word *statistics* is associated with the collection, analysis, interpretation, and presentation of numerical data. Sometimes, when presenting data, *notes* and *unit indicators* are included to help you interpret the data.

Headnotes give information about the table as a whole.

Public Elementary and Secondary School Enrollment, 1994–1998

[(in thousands) 44,111 represents 44,111,000.]
As of fall year, Kindergarten includes nursery schools.

Grade	1994	1995	1996	1997	1998, prel.
Pupils enrolled	44,111	44,840	45,611	46,127	46,535
Kindergarten and grades 1 to 8	31,898	32,341	32,764	33,073	33,344
Kindergarten	4047	4173	4202	4198	4171
First	3593	3671	3770	3755	3727
Second	3440	3507	3600	3689	3682
Third	3439	3445	3524	3597	3696
Fourth	3426	3431	3454	3507	3592
Fifth	3372	3438	3453	3458	3520
Sixth	3381	3395	3494	3492	3497
Seventh	3404	3422	3464	3520	3530
Eighth	3302	3356	3403	3415	3480
Unclassified ¹	494	502	401	442	460
Grades 9 to 12	12,213	12,500	12,847	13,054	13,191
Ninth	3604	3704	3801	3819	3856
Tenth	3131	3237	3323	3376	3382
Eleventh	2748	2826	2930	2972	3018
Twelfth	2488	2487	2586	2673	2724
Unclassified ¹	242	245	206	214	211

If the numerical data are too large, *unit indicators* are used to save space.

Footnotes give information about specific items within the table.

¹ Includes ungraded and special education.
Source: U.S. Census Bureau

Suppose you need to find the number of students enrolled in the 9th grade in 1997. The following steps can be used to determine this information.

- Step 1** Locate the number in the table. The number that corresponds to 1997 and 9th grade is 3819.
- Step 2** Determine the unit indicator. The *unit indicator* is thousands.
- Step 3** If the unit indicator is not 1 unit, multiply to find the data value. In this case, multiply 3819 by 1000.
- Step 4** State the data value. The number of students enrolled in the 9th grade in 1997 was 3,819,000.

Reading to Learn

Use the information in the table to answer each question.

1. Describe the data.
2. What information is given by the footnote?
3. How current is the data?
4. What is the unit indicator?
5. How many acres of state parks and recreation areas does New York have?
6. Which of the states shown had the greatest number of visitors? How many people visited that state's parks and recreation areas in 1999?

State	Acreage (1000)	Visitors (1000) ¹
United States	12,916	766,842
Alaska	3291	3855
California	1376	76,736
Florida	513	14,645
Indiana	178	18,652
New York	1016	61,960
North Carolina	158	13,269
Oregon	94	38,752
South Carolina	82	9563
Texas	628	21,446

Source: U.S. Census Bureau ¹ Includes overnight visitors.

Probability: Simple Probability and Odds

What You'll Learn

- Find the probability of a simple event.
- Find the odds of a simple event.

Why is probability important in sports?

A basketball player is at the free throw line. Her team is down by one point. If she makes an average of 75% of her free throws, what is the probability that she will tie the game with her first shot?



Vocabulary

- probability
- simple event
- sample space
- equally likely
- odds

PROBABILITY One way to describe the likelihood of an event occurring is with probability. The **probability** of a **simple event**, like a coin landing heads up when it is tossed, is a ratio of the number of favorable outcomes for the event to the total number of possible outcomes of the event. The probability of an event can be expressed as a fraction, a decimal, or a percent.

Suppose you wanted to find the probability of rolling a 4 on a die. When you roll a die, there are six possible outcomes, 1, 2, 3, 4, 5, or 6. This list of all possible outcomes is called the **sample space**. Of these outcomes, only one, a 4, is favorable. So, the probability of rolling a 4 is $\frac{1}{6}$, $0.1\bar{6}$, or about 16.7%.

Key Concept

Probability

The probability of an event a can be expressed as

$$P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Study Tip

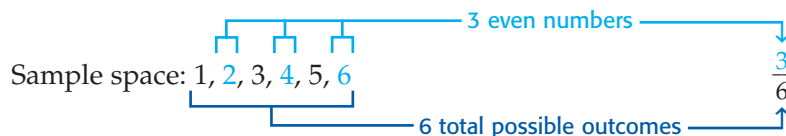
Reading Math

$P(a)$ is read the probability of a .

Example 1 Find Probabilities of Simple Events

- a. Find the probability of rolling an even number on a die.

There are six possible outcomes. Three of the outcomes are favorable. That is, three of the six outcomes are even numbers.



$$\text{So, } P(\text{even number}) = \frac{3}{6} \text{ or } \frac{1}{2}.$$

- b. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find $P(\text{blue})$.

There are 7 blue chips and 28 total chips.

$$\begin{aligned} P(\text{blue chip}) &= \frac{7}{28} && \leftarrow \text{number of favorable outcomes} \\ &= \frac{1}{4} \text{ or } 0.25 && \text{Simplify.} \end{aligned}$$

The probability of selecting a blue chip is $\frac{1}{4}$ or 25%.

- c. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find $P(\text{red or yellow})$.

There are 5 ways to pick a red chip and 6 ways to pick a yellow chip. So there are $5 + 6$ or 11 ways to pick a red or a yellow chip.

$$P(\text{red or yellow}) = \frac{11}{28} \quad \leftarrow \text{number of favorable outcomes}$$

$$\approx 0.39 \quad \text{Divide.}$$

The probability of selecting a red chip or a yellow chip is $\frac{11}{28}$ or about 39%.

- d. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find $P(\text{not green})$.

There are $5 + 7 + 6$ or 18 chips that are not green.

$$P(\text{not green}) = \frac{18}{28} \quad \leftarrow \text{number of favorable outcomes}$$

$$\approx 0.64 \quad \text{Divide.}$$

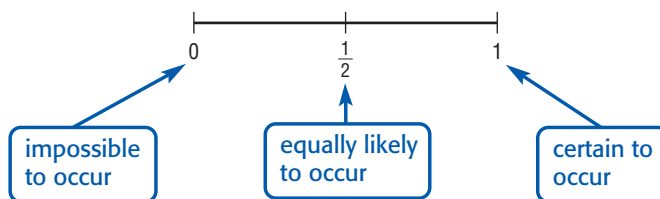
The probability of selecting a chip that is not green is $\frac{9}{14}$ or about 64%.

Study Tip

Reading Math

Inclusive means that the end values are included.

Notice that the probability that an event will occur is somewhere between 0 and 1 inclusive. If the probability of an event is 0, that means that it is impossible for the event to occur. A probability equal to 1 means that the event is certain to occur. There are outcomes for which the probability is $\frac{1}{2}$. When this happens, the outcomes are **equally likely** to occur or not to occur.



ODDS Another way to express the chance of an event occurring is with **odds**.

Key Concept

Odds

The odds of an event occurring is the ratio that compares the number of ways an event can occur (successes) to the number of ways it cannot occur (failures).

Study Tip

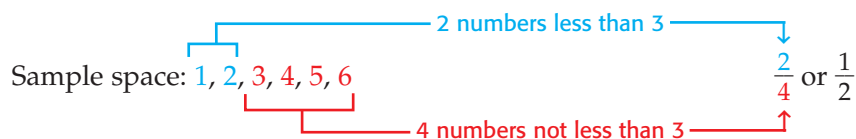
Odds

Odds are usually written in the form *number of successes* : *number of failures*.

Example 2 Odds of an Event

Find the odds of rolling a number less than 3.

There are 6 possible outcomes, 2 are successes and 4 are failures.



So, the odds of rolling a number less than three are $\frac{1}{2}$ or 1:2.



The odds *against* an event occurring are the odds that the event will *not* occur.

Study Tip

In this text, a *standard deck of cards* always indicates 52 cards in 4 suits of 13 cards each.

Example 3 Odds Against an Event

A card is selected at random from a standard deck of 52 cards. What are the odds against selecting a 3?

There are four 3s in a deck of cards, and there are $52 - 4$ or 48 cards that are not a 3.

odds against a 3 = $\frac{48}{4}$ ← number of ways to *not* pick a 3

The odds against selecting a 3 from a deck of cards are 12:1.

Example 4 Probability and Odds

WEATHER A weather forecast states that the probability of rain the next day is 40%. What are the odds that it will rain?

The probability that it will rain is 40%, so the probability that it will not rain is 60%.

odds of rain = 40:60 or 2:3

The odds that it will rain tomorrow are 2:3.

Check for Understanding

Concept Check

- OPEN ENDED** Give an example of an impossible event, a certain event, and an equally likely event when a die is rolled.
- Describe** how to find the odds of an event occurring if the probability that the event will occur is $\frac{3}{5}$.
- FIND THE ERROR** Mark and Doug are finding the probability of picking a red card from a standard deck of cards.

Mark

$$P(\text{red card}) = \frac{26}{26} \text{ or } \frac{1}{1}$$

Doug

$$P(\text{red card}) = \frac{26}{52} \text{ or } \frac{1}{2}$$

Who is correct? Explain your reasoning.

Guided Practice

A card is selected at random from a standard deck of cards. Determine each probability.

- $P(5)$
- $P(\text{odd number})$
- $P(\text{red } 10)$
- $P(\text{queen of hearts or jack of diamonds})$

Find the odds of each outcome if the spinner is spun once.

- multiple of 3
- even number less than 8
- odd number or blue
- red or yellow



Application

NUMBER THEORY One of the factors of 48 is chosen at random.

- What is the probability that the chosen factor is not a multiple of 4?
- What is the probability that the number chosen has 4 and 6 as two of its factors?

Practice and Apply

Homework Help

For Exercises	See Examples
14–35, 51, 54, 56	1
36–47, 52, 53, 55	2, 3
48, 49	4

Extra Practice

See page 824.

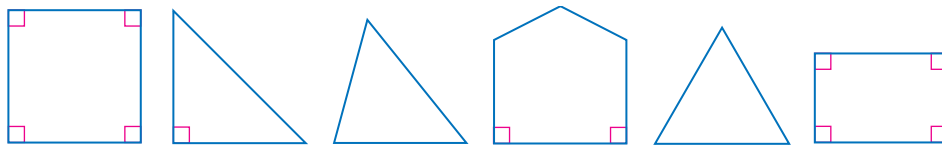
One coin is randomly selected from a jar containing 70 nickels, 100 dimes, 80 quarters, and 50 1-dollar coins. Find each probability.

14. $P(\text{quarter})$
15. $P(\text{dime})$
16. $P(\text{nickel or dollar})$
17. $P(\text{quarter or nickel})$
18. $P(\text{value less than } \$1.00)$
19. $P(\text{value greater than } \$0.10)$
20. $P(\text{value at least } \$0.25)$
21. $P(\text{value at most } \$1.00)$

Two dice are rolled, and their sum is recorded. Find each probability.

22. $P(\text{sum less than } 7)$
23. $P(\text{sum less than } 8)$
24. $P(\text{sum is greater than } 12)$
25. $P(\text{sum is greater than } 1)$
26. $P(\text{sum is between } 5 \text{ and } 10)$
27. $P(\text{sum is between } 2 \text{ and } 9)$

One of the polygons is chosen at random. Find each probability.



28. $P(\text{triangle})$
 29. $P(\text{pentagon})$
 30. $P(\text{not a triangle})$
 31. $P(\text{not a quadrilateral})$
 32. $P(\text{more than three sides})$
 33. $P(\text{more than one right angle})$
34. If a person's birthday is in April, what is the probability that it is the 29th?
 35. If a person's birthday is in July, what is the probability that it is after the 16th?

Find the odds of each outcome if a computer randomly picks a letter in the name *The United States of America*.

36. the letter *a*
37. the letter *t*
38. a vowel
39. a consonant
40. an uppercase letter
41. a lowercase vowel

More About . . .



Stamp Collecting

Stamp collecting can be a very inexpensive hobby. Most stamp collectors start by saving stamps from letters, packages, and postcards.

Source: United States Postal Service

STAMP COLLECTING Lanette collects stamps from different countries. She has 12 from Mexico, 5 from Canada, 3 from France, 8 from Great Britain, 1 from Russia, and 3 from Germany. Find the odds of each of the following if she accidentally loses one stamp.

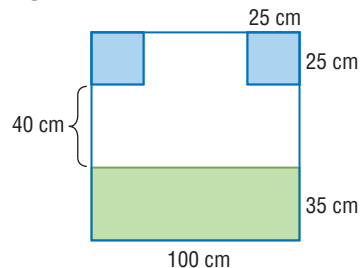
42. the stamp is from Canada
 43. the stamp is from Mexico
 44. the stamp is not from France
 45. the stamp is not from a North American country
 46. the stamp is from Germany or Russia
 47. the stamp is from Canada or Great Britain
48. If the probability that an event will occur is $\frac{3}{7}$, what are the odds that it will occur?
 49. If the probability that an event will occur is $\frac{2}{3}$, what are the odds against it occurring?



50. **CONTESTS** Every Tuesday, Mike's Submarine Shop has a business card drawing for a free lunch. Four coworkers from InvoAccounting put their business cards in the bowl for the drawing. If there are 80 cards in the bowl, what are the odds that one of the coworkers will win a free lunch?

GAMES For Exercises 51–53, use the following information.

A game piece is randomly placed on the board shown at the right by blindfolded players.



51. What is the probability that a game piece is placed on a shaded region?
52. What are the odds against placing a game piece on a shaded region?
53. What are the odds that a game piece will be placed within the green rectangle?

• **BASEBALL** For Exercises 54–56, use the following information.

The stem-and-leaf plot shows the number of home runs hit by the top major league baseball players in the 2000 season. **Source:** www.espn.com

Stem	Leaf
3	0 0 0 1 1 1 1 1 1 1 2 2 2 3
4	3 4 4 4 5 5 5 6 6 6 7 7 8 8 9
5	0 3 0 = 30

54. What is the probability that one of these players picked at random hit more than 35 home runs?
55. What are the odds that a randomly selected player hit less than 45 home runs?
56. If a player batted 439 times and hit 38 home runs, what is the probability that the next time the player bats he will hit a home run?

CONTESTS For Exercises 57 and 58, use the following information.

A fast-food restaurant is holding a contest in which the grand prize is a new sports car. Each customer is given a game card with their order. The contest rules state that the odds of winning the grand prize are 1:1,000,000.

57. For any randomly-selected game card, what is the probability that it is the winning game card for the grand prize?
58. Do your odds of winning the grand prize increase significantly if you have several game cards? Explain.

59. **CRITICAL THINKING** Three coins are tossed, and a tail appears on at least one of them. What is the probability that at least one head appears?

60. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is probability important in sports?

Include the following in your answer:

- examples of two sports in which probability is used and an explanation of each sport's importance, and
- examples of methods other than probability used to show chance.

More About . . .

Baseball

The record for the most home runs in a single season is 84. It was set by Joshua Gibson of the Homestead Grays in 1934.

Source: National Baseball Hall of Fame

WebQuest

You can use real-world data to find the probability that a person will live to be 100. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

61. If the probability that an event will occur is $\frac{12}{25}$, what are the odds that the event will *not* occur?
 (A) 12:13 (B) 13:12 (C) 13:25 (D) 25:12
62. What is the probability that a number chosen at random from the domain $\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$ will satisfy the inequality $3x + 2 \leq 17$?
 (A) 20% (B) 27% (C) 73% (D) 80%

Maintain Your Skills

Mixed Review

63. **WEATHER** The following data represents the average daily temperature in Fahrenheit for Sacramento, California, for two weeks during the month of May. Organize the data using a stem-and-leaf plot. (Lesson 2-5)
- | | | | | | | |
|------|------|------|------|------|------|------|
| 58.3 | 64.3 | 66.7 | 65.1 | 68.7 | 67.0 | 69.3 |
| 70.0 | 72.8 | 77.4 | 77.4 | 73.2 | 75.8 | 65.5 |

Evaluate each expression if $a = -\frac{1}{3}$, $b = \frac{2}{5}$, and $c = \frac{1}{2}$. (Lesson 2-4)

64. $b \div c$ 65. $2a \div b$ 66. $\frac{ab}{c}$

Find each sum. (Lesson 2-2)

67. $4.3 + (-8.2)$ 68. $-12.2 + 7.8$ 69. $-\frac{1}{4} + \left(-\frac{3}{8}\right)$ 70. $\frac{7}{12} + \left(-\frac{5}{6}\right)$

Find each absolute value. (Lesson 2-1)

71. $|4.25|$ 72. $|-8.4|$ 73. $\left|-\frac{2}{3}\right|$ 74. $\left|\frac{1}{6}\right|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression.
 (To review *evaluating expressions*, see Lesson 1-2.)

75. 6^2 76. 17^2 77. $(-8)^2$ 78. $(-11.5)^2$
 79. 1.6^2 80. $\left(\frac{5}{12}\right)^2$ 81. $\left(-\frac{4}{9}\right)^2$ 82. $\left(-\frac{16}{15}\right)^2$

Practice Quiz 2

Lessons 2-4 through 2-6

Find each quotient. (Lesson 2-4)

1. $-136 \div (-8)$ 2. $15 \div \left(-\frac{3}{8}\right)$ 3. $(-46.8) \div 4$

Simplify each expression. (Lesson 2-4)

4. $\frac{3a + 9}{3}$ 5. $\frac{4x + 32}{4}$ 6. $\frac{15n - 20}{-5}$

7. State the scale you would use to make a line plot for the following data. Then draw the line plot. (Lesson 2-5)

1.9 1.1 3.2 5.0 4.3 2.7 2.5 1.1 1.4 1.8 1.8 1.6
 4.3 2.9 1.4 1.7 3.6 2.9 1.9 0.4 1.3 0.9 0.7 1.9

Determine each probability if two dice are rolled. (Lesson 2-6)

8. $P(\text{sum of } 10)$ 9. $P(\text{sum} \geq 6)$ 10. $P(\text{sum} < 10)$



Algebra Activity

A Follow-Up of Lesson 2-6

Investigating Probability and Pascal's Triangle

Collect the Data

- If a family has one child, you know that the child is either a boy or a girl. You can make a simple table to show this type of family.

1 boy	1 girl
B	G

You can see that there are 2 possibilities for a one-child family.

- If a family has two children, the table below shows the possibilities for two children, including the order of birth. For example, BG means that a boy is born first and a girl second.

2 boys, 0 girls	1 boy, 1 girl	0 boys, 2 girls
BB	BG GB	GG

There are 4 possibilities for the two-child family: BB, BG, GB, or GG.

Analyze the Data

- Copy and complete the table that shows the possibilities for a three-child family.
- Make your own table to show the possibilities for a four-child family.
- List the total number of possibilities for a one-child, two-child, three-child, and four-child family. How many possibilities do you think there are for a five-child family? a six-child family? Describe the pattern of the numbers you listed.
- Find the probability that a three-child family has 2 boys and 1 girl.
- Find the probability that a four-child family has 2 boys and 2 girls.

3 boys	2 boys, 1 girl	1 boy, 2 girls	3 girls
BBB	BBG	BGG	GGG

Make a Conjecture

- Blaise Pascal was a French mathematician who lived in the 1600s. He is known for this triangle of numbers, called Pascal's triangle, although the pattern was known by other mathematicians before Pascal's time.

			1				Row 0
		1		1			Row 1
		1	2	1			Row 2
	1	3	3	1			Row 3
1	4	6	4	1			Row 4

Explain how Pascal's triangle relates to the possibilities for the make-up of families. (*Hint:* The first row indicates that there is 1 way to have 0 children.)

- Use Pascal's triangle to find the probability that a four-child family has 1 boy.

Square Roots and Real Numbers

What You'll Learn

- Find square roots.
- Classify and order real numbers.

Vocabulary

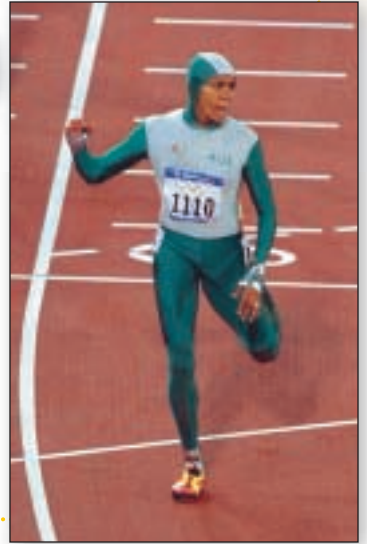
- square root
- perfect square
- radical sign
- principal square root
- irrational numbers
- real numbers
- rational approximations

How can using square roots determine the surface area of the human body?

In the 2000 Summer Olympics, Australian sprinter Cathy Freeman wore a special running suit that covered most of her body. The surface area of the human body may be found using the formula below, where height is measured in centimeters and weight is in kilograms.

$$\text{Surface Area} = \sqrt{\frac{\text{height} \times \text{weight}}{3600}} \text{ square meters}$$

The symbol $\sqrt{\quad}$ designates a square root.



SQUARE ROOTS A **square root** is one of two equal factors of a number. For example, one square root of 64 is 8 since $8 \cdot 8$ or 8^2 is 64. Another square root of 64 is -8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, whose square root is a rational number is called a **perfect square**.

The symbol $\sqrt{\quad}$, called a **radical sign**, is used to indicate a nonnegative or **principal square root** of the expression under the radical sign.

$$\sqrt{64} = 8 \quad \leftarrow \sqrt{64} \text{ indicates the } \textit{principal} \text{ square root of } 64.$$

$$-\sqrt{64} = -8 \quad \leftarrow -\sqrt{64} \text{ indicates the } \textit{negative} \text{ square root of } 64.$$

$$\pm\sqrt{64} = \pm 8 \quad \leftarrow \pm\sqrt{64} \text{ indicates } \textit{both} \text{ square roots of } 64.$$

Note that $-\sqrt{64}$ is not the same as $\sqrt{-64}$. The notation $-\sqrt{64}$ represents the negative square root of 64. The notation $\sqrt{-64}$ represents the square root of -64 , which is not a real number since no real number multiplied by itself is negative.

Example 1 Find Square Roots

Find each square root.

a. $-\sqrt{\frac{49}{256}}$

$-\sqrt{\frac{49}{256}}$ represents the negative square root of $\frac{49}{256}$.

$$\frac{49}{256} = \left(\frac{7}{16}\right)^2 \rightarrow -\sqrt{\frac{49}{256}} = -\frac{7}{16}$$

Study Tip

Reading Math

$\pm\sqrt{64}$ is read *plus or minus the square root of 64*.

b. $\pm\sqrt{0.81}$

$\pm\sqrt{0.81}$ represents the positive and negative square roots of 0.81.

$$0.81 = 0.9^2 \text{ and } 0.81 = (-0.9)^2$$

$$\pm\sqrt{0.81} = \pm 0.9$$

CLASSIFY AND ORDER NUMBERS Recall that rational numbers are numbers that can be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

As you have seen, the square roots of perfect squares are rational numbers.

However, numbers such as $\sqrt{3}$ and $\sqrt{24}$ are the square roots of numbers that are not perfect squares. Numbers like these cannot be expressed as a terminating or repeating decimal.

$$\sqrt{3} = 1.73205080\dots$$

$$\sqrt{24} = 4.89897948\dots$$

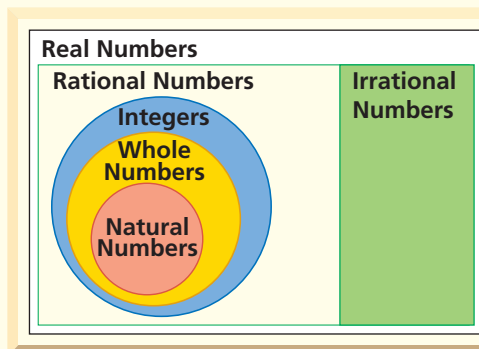
Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**.

Irrational numbers and rational numbers together form the set of **real numbers**.

Concept Summary

Real Numbers

The set of real numbers consists of the set of rational numbers and the set of irrational numbers.



Example 2 Classify Real Numbers

Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727\dots$ is a repeating decimal, this number is a rational number.

b. $\sqrt{121}$

Because $\sqrt{121} = 11$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

Because $\sqrt{56} = 7.48331477\dots$, which is not a repeating or terminating decimal, this number is irrational.

d. $-\frac{36}{4}$

Because $-\frac{36}{4} = -9$, this number is an integer and a rational number.

Study Tip

Common Misconception

Pay close attention to the placement of a negative sign when working with square roots. $\sqrt{-121}$ is undefined for real numbers since no real number multiplied by itself can result in a negative product.

In Lesson 2-1 you graphed rational numbers on a number line. However, the rational numbers alone do not complete the number line. By including irrational numbers, the number line is complete. This is illustrated by the **Completeness Property** which states that each point on the number line corresponds to exactly one real number.

Recall that inequalities like $x < 7$ are open sentences. To solve the inequality, determine what replacement values for x make the sentence true. This can be shown by the solution set {all real numbers less than 7}. Not only does this set include integers like 5 and -2 , but it also includes rational numbers like $\frac{3}{8}$ and $-\frac{12}{13}$ and irrational numbers like $\sqrt{40}$ and π .

Example 3 Graph Real Numbers

Graph each solution set.

a. $x > -2$



The heavy arrow indicates that all numbers to the right of -2 are included in the graph. The circle at -2 indicates -2 is *not* included in the graph.

b. $a \leq 4.5$



The heavy arrow indicates that all points to the left of 4.5 are included in the graph. The dot at 4.5 indicates that 4.5 is included in the graph.

To express irrational numbers as decimals, you need to use a rational approximation. A **rational approximation** of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number. For example, a rational approximation of $\sqrt{2}$ is 1.41 when rounded to the nearest hundredth.

Example 4 Compare Real Numbers

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

a. $\sqrt{19} \bullet 3.\bar{8}$

Find two perfect squares closest to $\sqrt{19}$ and write an inequality.

$$16 < 19 < 25 \quad \text{19 is between 16 and 25.}$$

$$\sqrt{16} < \sqrt{19} < \sqrt{25} \quad \text{Find the square root of each number.}$$

$$4 < \sqrt{19} < 5 \quad \text{\sqrt{19} is between 4 and 5.}$$

Since $\sqrt{19}$ is between 4 and 5, it must be greater than $3.\bar{8}$.
So, $\sqrt{19} > 3.\bar{8}$.

b. $7.\bar{2} \bullet \sqrt{52}$

You can use a calculator to find an approximation for $\sqrt{52}$.

$$\sqrt{52} = 7.211102551\dots$$

$$7.\bar{2} = 7.222\dots$$

Therefore, $7.\bar{2} > \sqrt{52}$.



You can write a set of real numbers in order from greatest to least or from least to greatest. To do so, find a decimal approximation for each number in the set and compare.

Example 5 Order Real Numbers

Write $2.\overline{63}$, $-\sqrt{7}$, $\frac{8}{3}$, $\frac{53}{-20}$ in order from least to greatest.

Write each number as a decimal.

$$2.\overline{63} = 2.6363636\dots \text{ or about } 2.636.$$

$$-\sqrt{7} = -2.64575131\dots \text{ or about } -2.646.$$

$$\frac{8}{3} = 2.6666666\dots \text{ or about } 2.667.$$

$$\frac{53}{-20} = -2.65$$

$$-2.65 < -2.646 < 2.636 < 2.667$$

The numbers arranged in order from least to greatest are $\frac{53}{-20}$, $-\sqrt{7}$, $2.\overline{63}$, $\frac{8}{3}$.

You can use rational approximations to test the validity of some algebraic statements involving real numbers.

Standardized Test Practice

A B C D

Example 6 Rational Approximation

Multiple-Choice Test Item

For what value of x is $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ true?

(A) $\frac{1}{2}$

(B) 0

(C) -2

(D) 3

Read the Test Item

The expression $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ is an open sentence, and the set of choices $\{\frac{1}{2}, 0, -2, 3\}$ is the replacement set.

Solve the Test Item

Replace x in $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ with each given value.

(A) $x = \frac{1}{2}$

$$\frac{1}{\frac{1}{\sqrt{2}}} \stackrel{?}{>} \sqrt{\frac{1}{2}} \stackrel{?}{>} \frac{1}{2} \quad \text{Use a calculator.}$$

$$1.41 > 0.71 > 0.5 \quad \checkmark \quad \text{True}$$

(B) $x = 0$

$$\frac{1}{\sqrt{0}} \stackrel{?}{>} \sqrt{0} \stackrel{?}{>} 0$$

False; $\frac{1}{\sqrt{0}}$ is not a real number.

(C) $x = -2$

$$\frac{1}{\sqrt{-2}} \stackrel{?}{>} \sqrt{-2} \stackrel{?}{>} -2$$

False; $\frac{1}{\sqrt{-2}}$ and $\sqrt{-2}$ are not real numbers.

(D) $x = 3$

$$\frac{1}{\sqrt{3}} \stackrel{?}{>} \sqrt{3} \stackrel{?}{>} 3 \quad \text{Use a calculator.}$$

~~$$0.58 > 1.73 > 3 \quad \text{False}$$~~

The inequality is true for $x = \frac{1}{2}$, so the correct answer is A.

The Princeton Review

Test-Taking Tip

You could stop when you find that A is a solution. But testing the other values is a good check.

Check for Understanding

Concept Check

- Tell whether the square root of any real number is *always*, *sometimes* or *never* positive. Explain your answer.
- OPEN ENDED** Describe the difference between rational numbers and irrational numbers. Give examples of both.
- Explain why you cannot evaluate $\sqrt{-25}$ using real numbers.

Guided Practice

Find each square root. If necessary, round to the nearest hundredth.

4. $-\sqrt{25}$ 5. $\sqrt{1.44}$ 6. $\pm\sqrt{\frac{16}{49}}$ 7. $\sqrt{32}$

Name the set or sets of numbers to which each real number belongs.

8. $-\sqrt{64}$ 9. $\frac{8}{3}$ 10. $\sqrt{28}$ 11. $\frac{56}{7}$

Graph each solution set.

12. $x < -3.5$ 13. $x \geq -7$

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

14. $0.3 \bullet \frac{1}{3}$ 15. $\frac{2}{9} \bullet 0.\bar{2}$ 16. $\frac{1}{6} \bullet \sqrt{6}$

Write each set of numbers in order from least to greatest.

17. $\frac{1}{8}, \sqrt{\frac{1}{8}}, 0.\bar{15}, -15$ 18. $\sqrt{30}, 5\frac{4}{9}, 13, \frac{1}{\sqrt{30}}$

Standardized Test Practice

A B C D

19. For what value of a is $-\sqrt{a} < -\frac{1}{\sqrt{a}}$ true?

- (A) $\frac{1}{3}$ (B) -4 (C) 2 (D) 1

Practice and Apply

Homework Help

For Exercises	See Examples
20–31, 50, 51	1
32–49	2
52–57	3
58–63	4
64–69	5

Extra Practice

See page 825.

Find each square root. If necessary, round to the nearest hundredth.

20. $\sqrt{49}$ 21. $\sqrt{81}$ 22. $\sqrt{5.29}$
 23. $\sqrt{6.25}$ 24. $-\sqrt{78}$ 25. $-\sqrt{94}$
 26. $\pm\sqrt{\frac{36}{81}}$ 27. $\pm\sqrt{\frac{100}{196}}$ 28. $\sqrt{\frac{9}{14}}$
 29. $\sqrt{\frac{25}{42}}$ 30. $\pm\sqrt{820}$ 31. $\pm\sqrt{513}$

Name the set or sets of numbers to which each real number belongs.

32. $-\sqrt{22}$ 33. $\frac{36}{6}$ 34. $\frac{1}{3}$
 35. $-\frac{5}{12}$ 36. $\sqrt{\frac{82}{20}}$ 37. $-\sqrt{46}$
 38. $\sqrt{10.24}$ 39. $\frac{-54}{19}$ 40. $-\frac{3}{4}$
 41. $\sqrt{20.25}$ 42. $\frac{18}{3}$ 43. $\sqrt{2.4025}$
 44. $\frac{-68}{35}$ 45. $\frac{6}{11}$ 46. $\sqrt{5.5696}$
 47. $\sqrt{\frac{78}{42}}$ 48. $-\sqrt{9.16}$ 49. π





Tourism

Built in 1758, the Sambro Island Lighthouse at Halifax Harbor is the oldest operational lighthouse in North America.

Source: Canadian Coast Guard

50. **PHYSICAL SCIENCE** The time it takes for a falling object to travel a certain distance d is given by the equation $t = \sqrt{\frac{d}{16}}$, where t is in seconds and d is in feet. If Krista dropped a ball from a window 28 feet above the ground, how long would it take for the ball to reach the ground?
51. **LAW ENFORCEMENT** Police can use the formula $s = \sqrt{24d}$ to estimate the speed s of a car in miles per hour by measuring the distance d in feet a car skids on a dry road. On his way to work, Jerome skidded trying to stop for a red light and was involved in a minor accident. He told the police officer that he was driving within the speed limit of 35 miles per hour. The police officer measured his skid marks and found them to be $43\frac{3}{4}$ feet long. Should the officer give Jerome a ticket for speeding? Explain.

Graph each solution set.

52. $x > -12$ 53. $x \leq 8$ 54. $x \geq -10.2$
 55. $x < -0.25$ 56. $x \neq -2$ 57. $x \neq \pm\sqrt{36}$

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

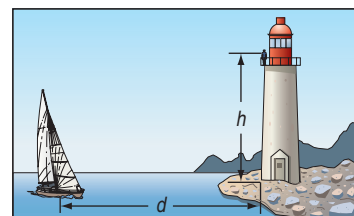
58. $5.\overline{72} \bullet \sqrt{5}$ 59. $2.\overline{63} \bullet \sqrt{8}$
 60. $\frac{1}{7} \bullet \frac{1}{\sqrt{7}}$ 61. $\frac{2}{3} \bullet \frac{2}{\sqrt{3}}$
 62. $\frac{1}{\sqrt{31}} \bullet \frac{\sqrt{31}}{31}$ 63. $\frac{\sqrt{2}}{2} \bullet \frac{1}{2}$

Write each set of numbers in order from least to greatest.

64. $\sqrt{0.42}, 0.\overline{63}, \frac{\sqrt{4}}{3}$ 65. $\sqrt{0.06}, 0.24, \frac{\sqrt{9}}{12}$
 66. $-1.\overline{46}, 0.2, \sqrt{2}, -\frac{1}{6}$ 67. $-4.\overline{83}, 0.4, \sqrt{8}, -\frac{3}{8}$
 68. $-\sqrt{65}, -6\frac{2}{5}, -\sqrt{27}$ 69. $\sqrt{122}, 7\frac{4}{9}, \sqrt{200}$

TOURISM For Exercises 70–72, use the following information.

The formula to determine the distance d in miles that an object can be seen on a clear day on the surface of a body of water is $d = 1.4\sqrt{h}$, where h is the height in feet of the viewer's eyes above the surface of the water.



70. A charter plane is used to fly tourists on a sightseeing trip along the coast of North Carolina. If the plane flies at an altitude of 1500 feet, how far can the tourists see?
71. Dillan and Marissa are parasailing while on vacation. Marissa is 135 feet above the ocean while Dillan is 85 feet above the ocean. How much farther can Marissa see than Dillan?
72. The observation deck of a lighthouse stands 120 feet above the ocean surface. Can the lighthouse keeper see a boat that is 17 miles from the lighthouse? Explain.
73. **CRITICAL THINKING** Determine when the following statements are all true for real numbers q and r .
- a. $q^2 > r^2$ b. $\frac{1}{q} < \frac{1}{r}$ c. $\sqrt{q} > \sqrt{r}$ d. $\frac{1}{\sqrt{q}} < \frac{1}{\sqrt{r}}$

Vocabulary and Concept Check

absolute value (p. 69)	irrational number (p. 104)	probability (p. 96)
additive inverses (p. 74)	line plot (p. 88)	radical sign (p. 103)
back-to-back stem-and-leaf plot (p. 89)	measures of central tendency (p. 90)	rational approximation (p. 105)
Completeness Property (p. 105)	natural number (p. 68)	rational number (p. 68)
coordinate (p. 69)	negative number (p. 68)	real number (p. 104)
equally likely (p. 97)	odds (p. 97)	sample space (p. 96)
frequency (p. 88)	opposites (p. 74)	simple event (p. 96)
graph (p. 69)	perfect square (p. 103)	square root (p. 103)
infinity (p. 68)	positive number (p. 68)	stem-and-leaf plot (p. 89)
integers (p. 68)	principal square root (p. 103)	whole number (p. 68)

State whether each sentence is *true* or *false*. If false, replace the underlined term or number to make a true sentence.

- The absolute value of -26 is 26.
- Terminating decimals are rational numbers.
- The principal square root of 144 is 12.
- $-\sqrt{576}$ is an irrational number.
- 225 is a perfect square.
- -3.1 is an integer.
- 0.666 is a repeating decimal.
- The product of two numbers with different signs is negative.

Lesson-by-Lesson Review

2-1 Rational Numbers on the Number Line

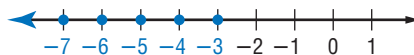
See pages
68–72.

Concept Summary

- A set of numbers can be graphed on a number line by drawing points.
- To evaluate expressions with absolute value, treat the absolute value symbols as grouping symbols.

Example

Graph $\{\dots, -5, -4, -3\}$.



The bold arrow means that the graph continues indefinitely in that direction.

Exercises Graph each set of numbers. See Example 2 on page 69.

9. $\{5, 3, -1, -3\}$

10. $\{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, \dots\}$

11. $\{\text{integers less than } -4 \text{ and greater than or equal to } 2\}$

Evaluate each expression if $x = -4$, $y = 8$, and $z = -9$. See Example 4 on page 70.

12. $32 - |y - 3|$

13. $3|x| - 7$

14. $4 + |z|$

15. $46 - y|x|$



2-2 Adding and Subtracting Rational Numbers

See pages
73–78.

Concept Summary

- To add rational numbers with the *same* sign, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different* signs, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.
- To subtract a rational number, add its additive inverse.

Examples

1 Find $-4 + (-3)$.

$$\begin{aligned} & -4 + (-3) \\ & = -(|-4| + |-3|) \quad \text{Both numbers are} \\ & = -(4 + 3) \quad \text{negative, so the} \\ & = -7 \quad \text{sum is negative.} \end{aligned}$$

2 Find $12 - 18$.

$$\begin{aligned} & 12 - 18 \\ & = 12 + (-18) \quad \text{To subtract 18,} \\ & = -(|-18| - |12|) \quad \text{add its inverse.} \\ & = -(18 - 12) \quad \text{The absolute value} \\ & = -6 \quad \text{of 18 is greater, so} \\ & \quad \text{the result is negative.} \end{aligned}$$

Exercises Find each sum or difference. See Examples 1–3 on pages 73–75.

16. $4 + (-4)$

17. $2 + (-7)$

18. $-0.8 + (-1.2)$

19. $-3.9 + 2.5$

20. $-\frac{1}{4} + \left(-\frac{1}{8}\right)$

21. $\frac{5}{6} + \left(-\frac{1}{3}\right)$

22. $-2 - 10$

23. $9 - (-7)$

24. $1.25 - 0.18$

25. $-7.7 - (-5.2)$

26. $\frac{9}{2} - \left(-\frac{1}{2}\right)$

27. $-\frac{1}{8} - \left(-\frac{2}{3}\right)$

2-3 Multiplying Rational Numbers

See pages
79–83.

Concept Summary

- The product of two numbers having the same sign is positive.
- The product of two numbers having different signs is negative.

Example

Multiply $\left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right)$.

$$\begin{aligned} \left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right) &= \frac{-15}{7} \cdot \frac{11}{3} \quad \text{Write as improper fractions.} \\ &= \frac{-55}{7} \text{ or } -7\frac{6}{7} \quad \text{Simplify.} \end{aligned}$$

Exercises Find each product. See Examples 1 and 3 on pages 79 and 80.

28. $(-11)(9)$

29. $12(-3)$

30. $-8.2(4.5)$

31. $-2.4(-3.6)$

32. $\frac{3}{4} \cdot \frac{7}{12}$

33. $\left(-\frac{1}{3}\right)\left(-\frac{9}{10}\right)$

Simplify each expression. See Example 2 on page 80.

34. $8(-3x) + 12x$

35. $-5(-2n) - 9n$

36. $-4(6a) - (-3)(-7a)$



2-4 Dividing Rational Numbers

See pages 84–87.

Concept Summary

- The quotient of two positive numbers is positive.
- The quotient of two negative numbers is positive.
- The quotient of a positive number and a negative number is negative.

Example

Simplify $\frac{-3(4)}{-2-3}$.

$$\begin{aligned} \frac{-3(4)}{-2-3} &= \frac{-12}{-2-3} && \text{Simplify the numerator.} \\ &= \frac{-12}{-5} && \text{Simplify the denominator.} \\ &= 2\frac{2}{5} && \text{same signs} \rightarrow \text{positive quotient} \end{aligned}$$

Exercises Find each quotient. See Examples 1–3 on pages 84 and 85.

37. $\frac{-54}{6}$

38. $-\frac{74}{8}$

39. $21.8 \div (-2)$

40. $-7.8 \div (-6)$

41. $-15 \div \left(\frac{3}{4}\right)$

42. $\frac{21}{24} \div \frac{1}{3}$

Simplify each expression. See Example 5 on page 85.

43. $\frac{14-28x}{-7}$

44. $\frac{-5+25x}{5}$

45. $\frac{-4x+24y}{4}$

Evaluate each expression if $x = -4$, $y = 2.4$, and $z = 3$. See Example 6 on page 85.

46. $xz - 2y$

47. $-2\left(\frac{2y}{z}\right)$

48. $\frac{2x-z}{4} + 3y$

2-5 Statistics: Displaying and Analyzing Data

See pages 88–94.

Concept Summary

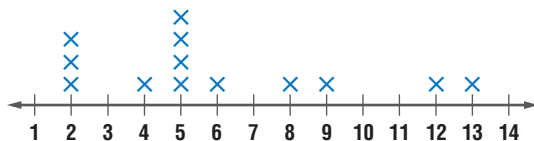
- A set of numerical data can be displayed in a line plot or stem-and-leaf plot.
- A measure of central tendency represents a centralized value of a set of data. Examine each measure of central tendency to choose the one most representative of the data.

Examples

1 Draw a line plot for the data.

2 8 6 4 5 9 13 12 5 2 5 5 2

The value of the data ranges from 2 to 13. Construct a number line containing those points. Then place an \times above a number each time it occurs.



- 2 SCHOOL** Melinda's scores on the 25-point quizzes in her English class are 20, 21, 12, 21, 22, 22, 22, 21, 20, 20, and 21. Which measure of central tendency best represents her grade?

mean: 20.2 Add the data and divide by 11.

median: 21 The middle value is 21.

mode: 21 The most frequent value is 21.

The median and mode are both representative of the data. The mean is less than most of the data.

Exercises

- 49.** Draw a line plot for the data. Then make a stem-and-leaf plot.

See Examples 1–3 on pages 88 and 89.

28	17	16	18	19	21	26	15
19	19	16	14	21	12	26	17
30	17	13	18	14	22	20	12
19	19	15	12	15	21	15	17

- 50. BUSINESS** Of the 42 employees at Pirate Printing, four make \$6.50 an hour, sixteen make \$6.75 an hour, six make \$6.85 an hour, thirteen make \$7.25 an hour, and three make \$8.85 an hour. Which measure best describes the average wage? Explain. See Examples 5 and 6 on pages 90 and 91.

- 51. HOCKEY** Professional hockey uses a point system based on wins, losses and ties, to determine teams' rank. The stem-and-leaf plot shows the number of points earned by each of the 30 teams in the National Hockey League during the 2000–2001 season. Which measure of central tendency best describes the average number of points earned? Explain.

See Example 5 on page 90.

Stem	Leaf
11	1 1 8
10	0 3 6 9
9	0 0 0 2 3 5 6 6 8
8	0 8 8
7	0 1 1 2 3
6	0 6 6 8
5	2 9 11 1 = 111

2-6

See pages 96–101.

Probability: Simple Probability and Odds

Concept Summary

- The probability of an event a can be expressed as
$$P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$
- The odds of an event can be expressed as the ratio of the number of successful outcomes to the number of unsuccessful outcomes.

Examples

- 1** Find the probability of randomly choosing the letter I in the word **MISSISSIPPI**.

$$P(\text{letter } I) = \frac{4}{11} \quad \leftarrow \text{number of favorable outcomes}$$

$$\approx 0.36 \quad \leftarrow \text{number of possible outcomes}$$

The probability of choosing an I is $\frac{4}{11}$ or about 36%.



- Extra Practice, see pages 823–825.
- Mixed Problem Solving, see page 854.

- 2 Find the odds that you will randomly select a letter that is *not* S in the word *MISSISSIPPI*.

number of successes : number of failures = 7:4

The odds of not selecting an S are 7:4.

Exercises Find the probability of each outcome if a computer randomly chooses a letter in the word *REPRESENTING*. See Example 1 on pages 96 and 97.

52. $P(S)$ 53. $P(E)$ 54. $P(\text{not } N)$ 55. $P(R \text{ or } P)$

Find the odds of each outcome if you randomly select a coin from a jar containing 90 pennies, 75 nickels, 50 dimes, and 30 quarters.

See Examples 2 and 3 on pages 97 and 98.

56. a dime 57. a penny 58. *not* a nickel 59. a nickel or a dime

2-7

Square Roots and Real Numbers

See pages
103–109.

Concept Summary

- A square root is one of two equal factors of a number.
- The symbol $\sqrt{\quad}$ is used to indicate the nonnegative square root of a number.

Example

Find $\sqrt{169}$.

$\sqrt{169}$ represents the square root of 169.

$$169 = 13^2 \rightarrow \sqrt{169} = 13$$

Exercises Find each square root. If necessary, round to the nearest hundredth.

See Example 1 on page 103.

60. $\sqrt{196}$ 61. $\pm\sqrt{1.21}$ 62. $-\sqrt{160}$ 63. $\pm\sqrt{\frac{4}{225}}$

Name the set or sets of numbers to which each real number belongs.

See Example 2 on page 104.

64. $\frac{16}{25}$ 65. $\frac{\sqrt{64}}{2}$ 66. $-\sqrt{48.5}$

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true. See Example 4 on page 105.

67. $\frac{1}{8} \bullet \frac{1}{\sqrt{49}}$ 68. $\sqrt{\frac{2}{3}} \bullet \frac{4}{9}$ 69. $\sqrt{\frac{3}{4}} \bullet \sqrt{\frac{1}{3}}$

70. **WEATHER** Meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$ to estimate the

amount of time t in hours a storm of diameter d will last. Suppose the eye of a hurricane, which causes the greatest amount of destruction, is 9 miles in diameter. To the nearest tenth of an hour, how long will the worst part of the hurricane last?

See Example 1 on pages 103 and 104.

Vocabulary and Concepts

Choose the correct term to complete each sentence.

- The (*absolute value, square*) of a number is its distance from zero on a number line.
- A number that can be written as a fraction where the numerator and denominator are integers and the denominator does not equal zero is a (*repeating, rational*) number.
- The list of all possible outcomes is called the (*simple event, sample space*).

Skills and Applications

Evaluate each expression.

- $-|x| - 38$ if $x = -2$
- $34 - |x + 21|$ if $x = -7$
- $-12 + |x - 8|$ if $x = 1.5$

Find each sum or difference.

- $-19 + 12$
- $-21 - (-34)$
- $16.4 + (-23.7)$
- $6.32 - (-7.41)$
- $-\frac{7}{16} + \frac{3}{8}$
- $-\frac{7}{12} - (-\frac{5}{9})$

Find each quotient or product.

- $-5(19)$
- $-56 \div (-7)$
- $96 \div (-0.8)$
- $(-7.8)(5.6)$
- $-\frac{1}{8} \div -5$
- $-\frac{15}{32} \div \frac{3}{4}$

Simplify each expression.

- $5(-3x) - 12x$
- $7(6h - h)$
- $-4m(-7n) + (3d)(-4c)$
- $\frac{36k}{4}$
- $\frac{9a + 27}{-3}$
- $\frac{70x - 30y}{-5}$

Find each square root. If necessary, round to the nearest hundredth.

- $-\sqrt{64}$
- $\sqrt{3.61}$
- $\pm\sqrt{\frac{16}{81}}$

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

- $\frac{1}{\sqrt{3}} \bullet \frac{1}{3}$
- $\sqrt{\frac{1}{2}} \bullet \frac{8}{11}$
- $\sqrt{0.56} \bullet \frac{\sqrt{3}}{2}$

STATISTICS For Exercises 31 and 32, use the following information.

The height, in inches, of the students in a health class are 65, 63, 68, 66, 72, 61, 62, 63, 59, 58, 61, 74, 65, 63, 71, 60, 62, 63, 71, 70, 59, 66, 61, 62, 68, 69, 64, 63, 70, 61, 68, and 67.

- Make a line plot of the data.
- Which measure of central tendency best describes the data? Explain.
- STANDARDIZED TEST PRACTICE** During a 20-song sequence on a radio station, 8 soft-rock, 7 hard-rock, and 5 rap songs are played at random. Assume that all of the songs are the same length. What is the probability that when you turn on the radio, a hard-rock song will be playing?

(A) $\frac{1}{4}$

(B) $\frac{7}{20}$

(C) $\frac{2}{5}$

(D) $\frac{13}{20}$

(E) $\frac{7}{10}$



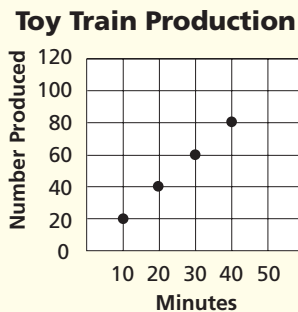
Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Darryl works 9 days at the State Fair and earns \$518.40. If he works 8 hours each day, what is his hourly pay? (Prerequisite Skill)
- (A) \$6.48 (B) \$7.20
(C) \$30.50 (D) \$57.60

2. The graph below shows how many toy trains are assembled at a factory at the end of 10-minute intervals. What is the best prediction for the number of products assembled per hour? (Prerequisite Skill)

- (A) 80
(B) 100
(C) 120
(D) 130



3. Which graph shows the integers greater than -2 and less than or equal to 3 ? (Lesson 2-1)

- (A)
- (B)
- (C)
- (D)



Test-Taking Tip

Question 1 If you don't know how to solve a problem, eliminate the answer choices you know are incorrect and then guess from the remaining choices. Even eliminating only one answer choice greatly increases your chance of guessing the correct answer.

4. Which number is the greatest? (Lesson 2-1)

- (A) $|-4|$ (B) $|4|$
(C) $|7|$ (D) $|-9|$

5. What is $-3.8 + 4.7$? (Lesson 2-2)

- (A) 0.9 (B) -0.9
(C) 8.5 (D) -8.5

6. Simplify $3(-2m) - 7m$. (Lesson 2-3)

- (A) $-12m$ (B) $-m$
(C) $-2m$ (D) $-13m$

7. Which statement about the stem-and-leaf plot is *not* true? (Lesson 2-5)

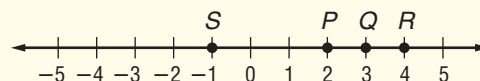
Stem	Leaf
3	1 1 5 6 8 8
4	2 2 2 4
5	0 0
6	0 3 7 8 9 9
7	4 7 4 = 74

- (A) The greatest value is 74.
(B) The mode is 42.
(C) Seven of the values are greater than 50.
(D) The least value is 38.

8. There are 4 boxes. If you choose a box at random, what are the odds that you will choose the one box with a prize? (Lesson 2-6)

- (A) 1:3 (B) 1:4
(C) 3:1 (D) 3:4

9. Which point on the number line is closest to $\sqrt{10}$? (Lesson 2-7)

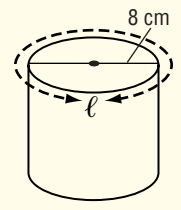


- (A) point P (B) point Q
(C) point R (D) point S

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Ethan needs to wrap a label around a jar of homemade jelly so that there is no overlap. Find the length of the label. (Prerequisite Skill)



11. Evaluate $\frac{5 - 1}{4 + 12 \div 3 \times 2}$. (Lesson 1-2)
12. Find the solution of $4m - 3 = 9$ if the replacement set is $\{0, 2, 3, 5\}$. (Lesson 1-3)
13. Write an algebraic expression for $2p$ plus three times the difference of m and n . (Lesson 1-6)
14. State the hypothesis in the statement *If $3x + 3 > 24$, then $x > 7$.* (Lesson 1-7)

Part 3 Quantitative Comparison

Compare the quantity in Column A to the quantity in Column B. Then determine whether:

- (A) the quantity in Column A is greater,
- (B) the quantity in Column B is greater,
- (C) the quantities are equal, or
- (D) the relationship cannot be determined from the information given.

Column A	Column B
$x > 0$	
$ x $	$ -x $

(Lesson 2-1)

Column A	Column B
$x > y > 0$	
$\frac{1}{x}$	$\frac{1}{y}$

(Lesson 2-5)

Column A	Column B
----------	----------

17. $\frac{1}{n} > 1$
- | | |
|---|-----|
| 1 | n |
|---|-----|
- (Lesson 2-6)
18. $a^2 = 49$
- | | |
|-----|---|
| a | 7 |
|-----|---|
- (Lesson 2-7)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

19. Mia has created the chart below to compare the three cellular phone plans she is considering. (Lessons 2-2 and 2-3)

Plan	Monthly Fee	Cost/Minute
A	\$5.95	\$0.30
B	\$12.95	\$0.10
C	\$19.99	\$0.08

- a. Write an algebraic expression that Mia can use to figure the monthly cost of each plan. Use C for the total monthly cost, m for the cost per minute, x for the monthly fee, and y for the minutes used per month.
- b. If Mia uses 150 minutes of calls each month, which plan will be least expensive? Explain.
20. The stem-and-leaf plot lists the annual profit for seven small businesses. (Lesson 2-5)

Stem	Leaf
3	2 9
4	1 1 3 5
5	0

- a. Explain how the absence of a key could lead to misinterpreting the data.
- b. How do the keys below affect how the data should be interpreted?
- $3|2 = 3.2$ $3|2 = 0.32$

